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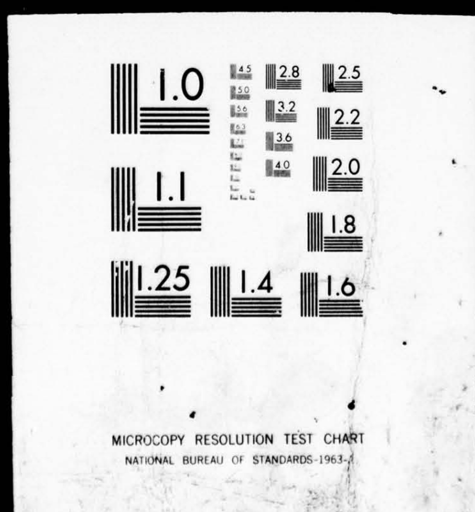

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(s,S) INVENTORY POLICIES FOR A WHOLESALE WAREHOUSE INVENTORY SYSTEM

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Decision Control Models in Operations Research

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Inventory managers often encounter erratic demand histories which are difficult to model. For example, periods of no demand are frequently observed, and when demand is positive, it tends to be quite large. Furthermore, periods of high demand are followed by several periods of low demand. One possible explanation for this sporadic and correlated demand behavior is that demand originates from separate facilities which employ (s,S) replenishment policies. That is, each period's demand is the sum

of the replenishment order quantities received from other facilities. In this investigation we consider the management of an inventory system exposed to this type of demand environment.

The complex nature of the demand history that arises in such a wholesale warehouse environment makes the computation of optimal replenishment policies prohibitive. We confine our attention, therefore, to stationary policies of the (s,S) form, and estimate the best values of the policy parameters by means of computer simulation.

In an applied setting it is impractical to use simulation to derive an inventory policy for each item of interest. We therefore seek an approximately optimal policy rule that is easy to compute. Two policy rules are examined in the wholesale warehouse environment. In order to discover the cost implications of ignoring the sporadic and correlated nature of the warehouse demand process, we first examine a stationary (s,S) policy rule that does not recognize these demand properties. For our experimental design, the policy rule's total cost performance is unsatisfactory; typically, 8% above that of stationary (s,S) policies found by simulation. The other policy rule we examine is an adaptation of the Power Approximation of Ehrhardt (1976) to a correlated demand environment. The only demand information required by the policy rule is the mean, variance, and variance over one leadtime of demand. For the same experimental design, total cost performance is typically only 3% above that of policies found by simulation.

FOREWORD

As part of the on-going research program in "Decision Control Models in Operations Research", Mr. Carl R. Schultz investigates the bahavior of multi-item inventory control systems at a warehouse level, in which demand is comprised of the aggregated replenishment orders from lower-echelon inventory control facilities. In this environment, the warehouse demand probability distributions are sporadic and exhibit correlation from one time period to the next. Under an assumption of limited demand information, Mr. Schultz adapts the Power Approximation (Technical Report #7), which was originally designed for independent demand distributions, for warehouse replenishment decisions. Several sections of this report parallel similar findings in earlier reports. Other related reports dealing with the research program are listed as follows.

Harvey M. Wagner
Principal Inventigator

Richard Ehrhardt
Co-principal Inventigator

MacCormick, A. (1975), Statistical Problems in Inventory Control, ONR and ARO Technical Report, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 1), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, School of Organization and Management, Yale University, 58 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J. G. (1976), The Power Approximation: Control of Multi-item Inventory Systems with Constant Standard-deviation-to-mean Ratio for Demand, ONR and ARO Technical Report 10, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R. L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ARO Technical Report 11, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

Schultz, C. R., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ONR and ARO Technical Report 13, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

CONTENTS

ABSTRACT

Inventory managers often encounter erratic demand histories which are difficult to model. For example, periods of no demand are frequently observed, and when demand is positive, it tends to be quite large. Furthermore, periods of high demand are followed by several periods of low demand. One possible explanation for this sporadic and correlated demand behavior is that demand originates from separate facilities which employ (s,S) replenishment policies. That is, each period's demand is the sum of the replenishment order quantities received from other facilities. In this investigation we consider the management of an inventory system exposed to this type of demand environment.

The complex nature of the demand history that arises in such a wholesale warehouse environment makes the computation of optimal replenishment policies prohibitive. We confine our attention, therefore, to stationary policies of the (s,S) form, and estimate the best values of the policy parameters by means of computer simulation.

In an applied setting it is impractical to use simulation to derive an inventory policy for each item of interest. We therefore seek an approximately optimal policy rule that is easy to compute. Two policy rules are examined in the wholesale warehouse environment. In order to discover the cost implications of ignoring the sporadic and correlated nature of the warehouse demand process, we first examine a stationary (s,S) policy rule that does not recognize these demand properties. For our experimental design, the policy rule's total cost performance is

unsatisfactory; typically, 8% above that of stationary (s,S) policies found by simulation. The other policy rule we examine is an adaptation of the Power Approximation of Ehrhardt (1976) to a correlated demand environment. The only demand information required by the policy rule is the mean, variance, and variance over one leadtime of demand. For the same experimental design, total cost performance is typically only 3% above that of policies found by simulation.

1. (s,S) INVENTORY POLICIES FOR A WHOLESALE WAREHOUSE SYSTEM

In many inventory systems one often observes an erratic demand history whose underlying generating process is difficult to explain. For example, consider a situation in which periods of no demand are frequently observed, and when demand is positive, it tends to be quite large. One possible explanation for this behavior is that demand originates from other facilities which employ (s,S) replenishment policy rules. If this is true, demand may also be correlated from one period to another. We consider the use of (s,S) replenishment policies for an erratic, correlated demand history that arises in a wholesale warehouse environment. We analyze a two-echelon inventory system consisting of a number of lower-echelon facilities (stores) satisfying exogenous customer demand and themselves acting as customers to a single upper-echelon facility (warehouse).

Most, if not all, of the research associated with the stochastic, two-echelon inventory control problem [Clark (1972)], has been in the direction of establishing rules or policies which, when applied at the individual facilities, satisfy a prescribed system-wide performance objective, such as minimal expected cost. The complexity of centralized system control makes the problem of determining optimal order and supply policies computationally untractable unless very restrictive assumptions are imposed on both the model and the policy forms, assumptions which tend to be over-restrictive relative to practical applications. In addition, centralized control over the entire system is often not possible in applied situations. In this investigation we depart from the tradi-

tional approach of system-wide inventory control and instead view the management of the warehouse simply as a single-facility control problem. The demand process at the warehouse is given by the aggregated replenishment processes of the stores which, we shall assume, employ (s,S) policies. As noted earlier, this demand process can be highly erratic and significantly correlated, unlike the demand processes commonly studies in the single-facility inventory control problem. In fact, little research has dealt with the case of dependent demands, and existing results [Karlin and Fabens (1959)] are not applicable to this demand structure.

1.1 A Wholesale Warehouse Inventory Model

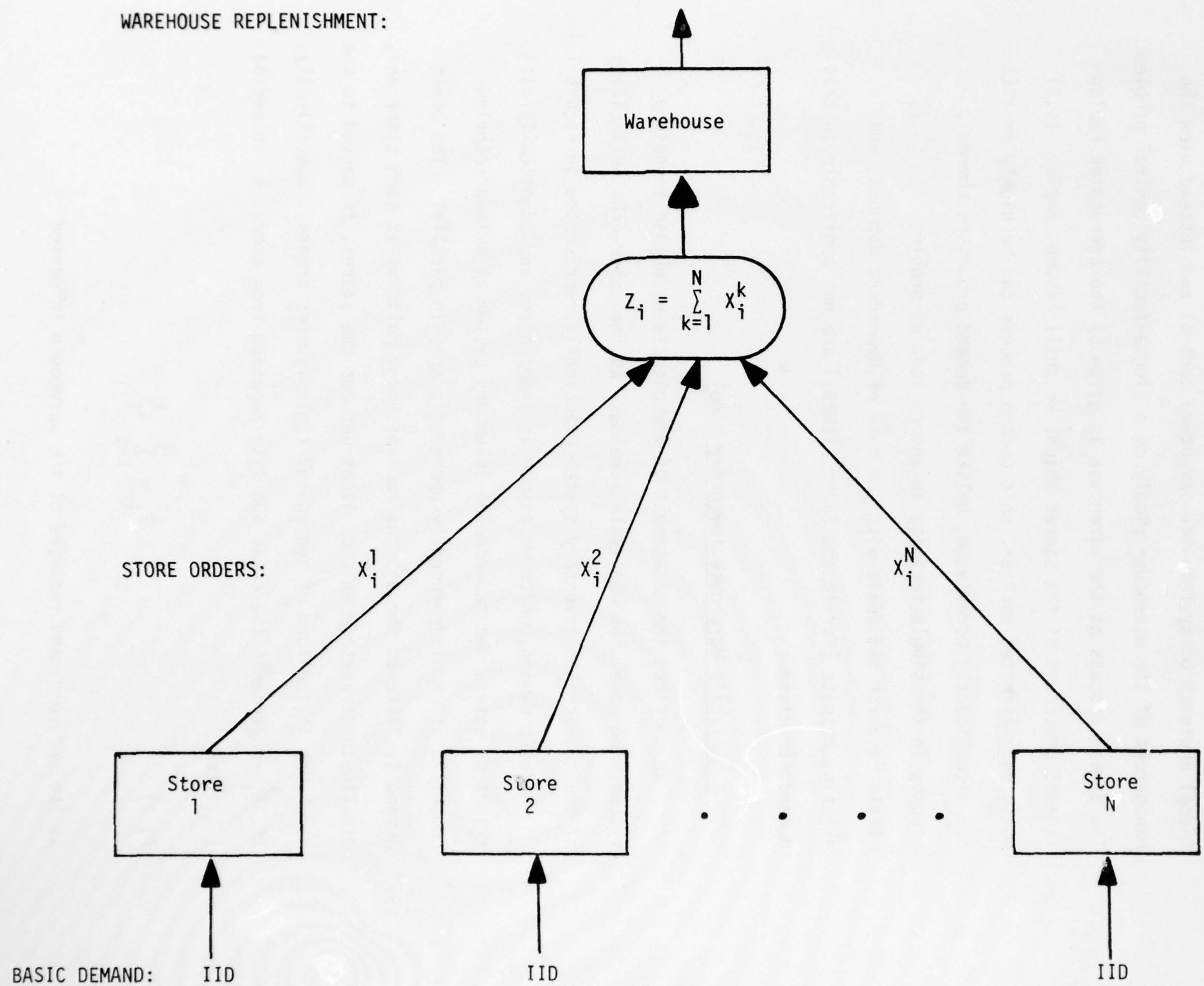
We consider the management of a wholesale warehouse inventory system. We model the wholesale warehouse as the upper-echelon facility of the two-echelon inventory system graphically represented in Figure 1.1 .

Basic demand, which we assume is independent and identically distributed, enters the two-echelon inventory system at a lower-echelon comprised of N independently-operated stores in parallel. The basic demand is filtered through the replenishment policies at each store and, disallowing transshipments of items between the stores, is passed to the warehouse in the form of aggregated replenishment orders. Specifically, if X_i^k represents the order quantity received from store k in period i , then

$$Z_i = \sum_{k=1}^{N} X_i^k \tag{1.1}$$

is the entire demand realized at the warehouse in period i .

FIGURE 1.1 A TWO-ECHELON INVENTORY SYSTEM



We postulate a single-item, periodic review inventory model for each facility in the two-echelon system. We assume that demand at each facility is met as long as there is stock on hand, and when a stockout occurs, unfilled demand is backlogged until sufficient replenishments arrive. Items kept in inventory are assumed to be conserved, there being no losses by deterioration, obsolescence, or pilferage. Inventory on hand at the end of a current period is the inventory from the previous period plus any replenishment that arrives, less demand in the current period. If inventory on hand is negative, its absolute value is the amount of backlogged demand. The time sequence of events within any period is taken to be order, delivery, and demand.

The cost structure at the warehouse is of a simple form. At the end of each period a unit holding cost h or a unit penalty cost p is incurred for each unit on hand or on back order, respectively. The cost of a replenishment quantity is assumed to be linear with a fixed setup cost K, and warehouse replenishments are assumed to be delivered a fixed lead time L after being ordered. The criterion for optimal inventory control is minimization of the undiscounted expected cost per period over an infinite horizon.

We assume that control over replenishment at each facility in the two-echelon system is exercised by an (s,S) policy: whenever inventory x on hand and on order is less than or equal to the value s, an order is placed for a replenishment of size $S - x$.

Under the model assumptions described above, an (s,S) policy is known to be optimal if demands are independent and identically distributed

[Iglehart (1963a, b)] , and methods for computing optimal policies have been developed [Veinott and Wagner (1965)] . When demands in successive periods are dependent, as is the case at the warehouse, an optimal policy will not be of the (s,S) form. Nevertheless, in this study we confine our attention to warehouse replenishment policies of the (s,S) form, since their simple form has led to their frequent use in applied situations.

1.2 An Algorithm to Set (s,S) With Partial Knowledge

We base our algorithm to determine values for the warehouse policy parameters s and S on the Power Approximation of Ehrhardt (1976) . The Power Approximation is an algorithm for computing approximately optimal values for (s,S) using only the mean μ and variance σ^2 of demand and is based on asymptotic renewal theory. Furthermore, the Power Approximation is derived for independent, identically distributed demands. One of the goals of this study is to investigate an adaptation of the Power Approximation to the sporadic and correlated demand structure present in our wholesale warehouse inventory system.

The Power Approximation is executed as follows. Let

$$D_p = (1.463)\mu^{.364}(K/h)^{.498}(\sigma_{L+1})^{.1382} \tag{1.2}$$

and

$$s_p = (L + 1)\mu + (\sigma_{L+1})^{.832}(\sigma^2/\mu)^{.187}(.220/z + 1.142 - 2.866z) , \tag{1.3}$$

where

(1.4) $$z = \{D_p/[(1 + p/h)\sigma_{L+1}]\}^{.5} \ ,$$

and

(1.5) $$\sigma^2_{L+1} = (L + 1)\sigma^2 \ .$$

If D_p/μ is greater than 1.5 , let $s = s_p$ and $S = s_p + D_p$. Otherwise compute

(1.6) $$S_0 = (L + 1)\mu + v\sigma_{L+1} \ ,$$

where v is the solution to

(1.7) $$\int_{-\infty}^{v} (2\pi)^{-\frac{1}{2}} \exp(-x^2/2)\, dx = p/(p + h) \ .$$

The policy parameters are given by

(1.8) $$s = \text{minimum}(s_p, S_0)$$
$$S = \text{minimum}(S_p, S_0) \ .$$

If demands are integer-valued, then s_p , D_p , and S_0 are rounded to the nearest integer.

Since the Power Approximation is designed for independent, identically distributed demands, the term σ^2_{L+1} given by (1.5) represents the variance of demand over L+1 periods. Our adaptation of the Power Approximation adjusts this term to account for correlation in the demand process.

If the demand process is covariance-stationary, then the variance σ^2_{L+1} of demand over $L+1$ periods can be written as

$$\sigma^2_{L+1} = [(L + 1) + 2 \sum_{j=1}^{L} (L + 1 - j)\rho(j)]\sigma^2 , \tag{1.9}$$

where $\rho(j)$ is the correlation between demands separated by j periods. Thus our adaptation of the power approximation is the replacement of (1.5) with (1.9) .

In Section 2 we present a detailed analysis of the demand process at the warehouse. Expressions for the mean, variance, and autocorrelation function of the warehouse demand process, in terms of the store parameters, are derived.

In Section 3 we discuss several cost-effective policy rules for the wholesale warehouse. The complexity of the warehouse demand process makes the computation of optimal policies prohibitive. We confine our attention to policies of the (s,S) form and use simulation to estimate the minimum-cost (s,S) policy for use in the warehouse environment. The simulation program which finds these "best" (s,S) policies is also described in Section 3 . Finally, an experimental design is presented for the evaluation, in later sections, of approximately-optimal policy rules. The performance of these policy rules in the wholesale warehouse environment is evaluated by comparing their key operating characteristics with the corresponding operating characteristics of best (s,S) policies under the same conditions.

In Section 4 an evaluation of policy rules that ignore the sporadic

and correlated properties of the warehouse demand process is presented. In particular, these are policy rules that would be optimal if warehouse demands were independently and identically distributed with a negative binomial distribution. We find the performance of these policies to be unsatisfactory. Typically, for a large multi-item system, expected total costs are approximately 8% higher than those associated with the simulation-derived best (s,S) policies.

In Section 5 an evaluation of the Adjusted Power Approximation, discussed above, is presented. We find that by adjusting for correlation, significant improvement can result. Specifically, expected total costs are about 3% above best values for the same multi-item system.

Section 6 concludes this investigation with a summary of our findings, conclusions, and suggestions for future research.

2. ANALYSIS OF THE WAREHOUSE DEMAND PROCESS

In this section we present a detailed analysis of the demand environment in the wholesale warehouse inventory model described in Section 1.

In Subsection 2.1 we study the replenishment-quantity process of a single store employing a stationary (s,S) policy and experiencing independent, identically distributed, integer-valued, non-negative demands. In particular, we develop the stationary distribution of the process, and derive simple expressions for its mean and variance. We also present a recursive formula for the autocorrelation function of the process.

Finally, in Subsection 2.2 we make use of our model assumptions to derive simple expressions for the mean, variance and autocorrelation function of the warehouse demand process in terms of their counterparts for the single store replenishment-quantity process.

2.1 Properties of the Replenishment-Quantity Process at a Single Store

We assume the demand process at a store, denoted by $q_1, q_2, \ldots$, is a sequence of non-negative, integer-valued, independent and identically distributed random variables having cumulative distribution $\Phi(\cdot)$ and probability mass function $\phi(\cdot)$. Let $\Phi^n(\cdot)$ and $\phi^n(\cdot)$ be, for $n \geq 1$, the n-fold convolutions of $\Phi(\cdot)$ and $\phi(\cdot)$. Let $\Phi^0(\cdot)$ represent the distribution whose full mass is located at zero. That is,

$$\phi^0(y) = \begin{cases} 0 & y < 0 \\ 1 & y \geq 0 \end{cases} \quad , \tag{2.1}$$

which implies that

$$\phi^0(y) = \begin{cases} 0 & y \neq 0 \\ 1 & y = 0 \end{cases} \quad . \tag{2.2}$$

We define the renewal functions

$$M(y) = \sum_{k=1}^{\infty} \phi^k(y) \ , \tag{2.3}$$

and

$$m(y) = \sum_{k=1}^{\infty} \phi^k(y) \quad . \tag{2.4}$$

Let μ_s and σ_s^2 denote, respectively, the mean and variance of the store's demand distribution.

We assume that the store employs a stationary (s,S) replenishment policy. Let $D = S - s$ and note that the possible replenishment quantity values are $0, D+1, D+2, \ldots$.

Using the store assumptions given above, we analyze the properties of the store's replenishment-quantity process. Several important results from this analysis are given by the following lemma.

<u>Lemma 2.1</u>:

If X_i represents the replenishment order quantity in period i, then

i) the stationary distribution of X_i is given by

$$Pr[X_i = 0] = M(D)/[1 + M(D)] ,$$

and for $k = 0,1, 2, \ldots$,

$$Pr[X_i = D+k+1] = [\phi(D+k+1) + \sum_{j=0}^{D} \phi(j+k+1)m(D-j)]/[1 + M(D)] ,$$

ii) the expected value of X_i, denoted by μ_r, satisfies

$$\mu_r = \mu_s$$

iii) *the variance of* X_i *denoted by* σ_r^2, *satisfies*

$$\sigma_r^2 = \sigma_s^2 + [2\mu_s \sum_{k=0}^{D} km(k)]/[1 + M(D)] ,$$

and

iv) the correlation between replenishment order quantities separated by j periods, denoted by $\rho_r(j)$, satisfies the recursive relationship

$$\rho_r(j) = \begin{cases} -[\mu_r \sum_{k=0}^{D} k\phi(k)]/\sigma_r^2 & j = 1 \\ -[\mu_r \sum_{k=0}^{D} k\phi^j(k)]/\sigma_r^2 - \sum_{\ell=1}^{j-1} \phi^{j-\ell}(D)\rho_r(\ell) & j > 1 . \end{cases}$$

Proof:

Let X_i represent the replenishment order quantity in period i. We proceed to derive an expression for the distribution of X_i conditioned on the number of periods n since the last order was placed.

First, consider the case when an order was placed in the preceding period, that is, $n = 1$. Then for $0 \le t \le D$

$$\text{(2.5)} \qquad \begin{aligned} \Pr[X_i \le t | n = 1] &= \Pr[X_i = 0 | n = 1] \\ &= \Pr[q_1 \le D] \\ &= \Phi(D) , \end{aligned}$$

and for $t > D$,

$$\text{(2.6)} \qquad \begin{aligned} \Pr[X_i \le t | n = 1] &= \Pr[q_1 \le t] \\ &= \Phi(t) . \end{aligned}$$

Now suppose $n > 1$. For $0 \le t \le D$ we have

$$\text{(2.7)} \qquad \begin{aligned} \Pr[X_i \le t | n = j] &= \Pr[X_i = 0 | n = j] \\ &= \Pr[q_1+q_2+\ldots+q_j \le D | q_1+q_2+\ldots+q_{j-1} \le D] \\ &= \Phi^j(D)/\Phi^{j-1}(D) . \end{aligned}$$

For $t > D$, we have

$$\text{(2.8)} \qquad \begin{aligned} \Pr[X_i \le t | n = j] &= \Pr[X_i \le D | n = j] + \Pr[D < X_i \le t | n=j] \\ &= \{\Phi^j(D) + \Pr[D < q_1+q_2+\ldots+q_j \le t \\ &\qquad \text{and } q_1+q_2+\ldots+q_{j-1} \le D]\}/\Phi^{j-1}(D) . \end{aligned}$$

Observe that

$$(2.9)\qquad \Pr[D < q_1+q_2+\ldots+q_j \le t \text{ and } q_1+q_2+\ldots+q_{j-1} \le D]$$

$$= \Pr[q_1+q_2+\ldots+q_{j-1} \le \min(D,t-q_j)] - \Pr[q_1+q_2+\ldots+q_{j-1} \le D-q_j] .$$

Conditioning on the value of q_j, the right-hand side of (2.9) becomes

$$(2.10)\qquad \Phi^{j-1}(D)\Phi(t-D-1) + \sum_{k=t-D}^{t} \Phi^{j-1}(t-k)\phi(k) - \Phi^{j}(D) .$$

Therefore,

$$(2.11)\qquad \Pr[X_i \le t|n=j] = \Phi(t-D-1) + \left[\sum_{k=t-D}^{t} \Phi^{j-1}(t-k)\phi(k)\right]/\Phi^{j-1}(D) .$$

To summarize, for $j = 1,2,3,\ldots$,

$$(2.12)\qquad \Pr[X_i \le t|n=j] = \begin{cases} 0 & t < 0 \\ \Phi^{j}(D)/\Phi^{j-1}(D) & 0 \le t \le D \\ \Phi(t-D-1) + \left[\sum_{k=t-D}^{t} \Phi^{j-1}(t-k)\phi(k)\right]/\Phi^{j-1}(D) & t > D . \end{cases}$$

Using (2.12) we can readily obtain the following conditional probabilities:

$$(2.13)\qquad \Pr[X_i = 0|n=j] = \Phi^{j}(D)/\Phi^{j-1}(D) , \qquad j = 1,2,3,\ldots ,$$

and

$$(2.14)\quad Pr[X_i = D+m+1 | n = j] = \left[\sum_{k=0}^{D} \phi(k+m+1)\phi^{j-1}(D-k)\right] / \Phi^{j-1}(D) ,$$

$$m = 0,1,2,\ldots \quad .$$

Using a Markov chain analysis, we can determine the stationary or limiting distribution of the number of periods n between orders. Specifically,

$$(2.15)\qquad Pr[n = j] = \Phi^{j-1}(D)/[1 + M(D)] , \qquad j = 1,2,3,\ldots \quad .$$

The limiting probabilities given in (2.15) enable us to explicitly determine the stationary distribution of *the single period replenishment* quantity X_i . For $m = 0,1,2,\ldots$, we have

$$(2.16)\quad Pr[X_i = D+m+1] = \sum_{j=1}^{\infty} Pr[X_i = D+m+1 | n = j] \; Pr[n = j] \quad .$$

Using (2.14) we can express the right-hand side of (2.16) as

$$(2.17)\qquad \left[\sum_{j=1}^{\infty} \sum_{k=0}^{D} \phi(k+m+1)\phi^{j-1}(D-k)\right] / [1 + M(D)]$$

$$= \left[\phi(D+m+1) + \sum_{k=0}^{D} \phi(k+m+1) \sum_{j=1}^{\infty} \phi^{j}(D-k)\right] / [1 + M(D)] \quad .$$

Thus

$$(2.18)\quad Pr[X_i = D+k+1] = \left[\phi(D+k+1) + \sum_{j=0}^{D} \phi(j+k+1)m(D-j)\right] / [1 + M(D)] ,$$

$$k = 0,1,2,\ldots \quad .$$

In a similar fashion we obtain

$$(2.19)\qquad Pr[X_i = 0] = M(D)/[1 + M(D)] \quad .$$

Thus part i has been established.

Since all demand eventually finds its way into replenishment orders, we would expect the mean μ_r of the stationary replenishment distribution, given by (2.18) and (2.19), to be the same as the mean μ_s of the store's demand distribution. This intuitive result can be shown directly using the stationary replenishment distribution. The proof, however, is not presented here.

As part ii of the lemma indicates, the mean μ_r of the stationary replenishment distribution does not depend on the store's policy parameters s and S. As one might well anticipate, however, the variance σ_r^2 of the stationary replenishment distribution depends upon D.

Let X_i represent the single period replenishment quantity. Then from part ii of the lemma

(2.20)
$$\begin{aligned} \text{Var } X_i &= EX_i^2 - (EX_i)^2 \\ &= EX_i^2 - \mu_s^2 \quad . \end{aligned}$$

We, therefore, need to evaluate

(2.21)
$$EX_i^2 = \sum_{n=0}^{\infty} (D+n+1)^2 \Pr[X_i = D+n+1] \quad .$$

Using (2.18) we have

(2.22)
$$EX_i^2 = \left[\sum_{n=0}^{\infty} (D+n+1)^2 \phi(D+n+1) + \sum_{y=0}^{D} m(D-y) \sum_{n=0}^{\infty} (D+n+1)^2 \phi(y+n+1)\right] / [1 + M(D)] \quad .$$

Observe that

$$(2.23) \qquad \sum_{n=0}^{\infty} (D+n+1)^2 \phi(D+n+1) = \sigma_s^2 + \mu_s^2 - \sum_{k=0}^{D} k^2 \phi(k) \quad ,$$

where σ_s^2 is the variance of the store's demand distribution. Also

$$(2.24) \qquad \sum_{n=0}^{\infty} (D+n+1)^2 \phi(y+n+1) = \sum_{n=0}^{\infty} [(D-y)^2 + 2(D-y)(y+n+1) + (y+n+1)^2] \cdot \phi(y+n+1)$$

$$= (D-y)^2 + \sigma_s^2 + \mu_s^2 + 2\mu(D-y) - \sum_{k=0}^{y} (y-D-k)^2 \cdot \phi(k) \quad .$$

With the aid of (2.23) , (2.24) , and the fact that

$$(2.25) \qquad M(z) = \sum_{j=0}^{z} m(j) \quad ,$$

we can rewrite (2.22) as

$$(2.26) \qquad EX_i^2 = \sigma_s^2 + \mu_s^2 + [2\mu_s \sum_{k=0}^{D} km(k)]/[1 + M(D)]$$

$$+ \left\{ \sum_{y=0}^{D} m(D-y)[(D-y)^2 - \sum_{k=0}^{y} (y-D-k)^2 \phi(k)] - \sum_{k=0}^{D} k^2 \phi(k) \right\} / [1 + M(D)] \ .$$

Rearranging terms, we have

$$(2.27) \qquad \sum_{y=0}^{D} m(D-y)[(D-y)^2 - \sum_{k=0}^{y} (y-D-k)^2 \phi(k)] - \sum_{k=0}^{D} k^2 \phi(k)$$

$$= \sum_{y=0}^{D} (D-y)^2 [m(D-y) - \sum_{k=0}^{y} \phi(k) m(D-y-k) - \phi(D-y)]$$

which is zero, since $m(\cdot)$ satisfies the renewal equations

$$(2.28) \qquad m(z) = \phi(z) + \sum_{y=0}^{z} \phi(y)m(z-y) \qquad \text{for} \quad z \ge 0 \quad .$$

Therefore,

$$(2.29) \qquad \sigma_r^2 = \sigma_s^2 + [2\mu_s \sum_{k=0}^{D} km(k)]/[1 + M(D)] \quad ,$$

and part iii of the lemma has been established. It can also be shown, as one would expect, that σ_r^2 is increasing in D .

We previously observed that due to the nature of (s,S) policies the replenishment process $\{X_i; \; i \ge 1\}$ will exhibit autocorrelated behavior. Since the replenishment process is covariance-stationary, the covariance between X_i and X_{i+j} will be independent of i . The autocovariance of the replenishment process at lag j , denoted $\gamma_r(j)$, is defined as

$$(2.30) \qquad \gamma_r(j) \equiv \mathrm{Cov}(X_i, X_{i+j}) = E[(X_i - \mu_r)(X_{i+j} - \mu_r)]$$
$$= EX_iX_{i+j} - \mu_r^2 \quad .$$

We have

$$(2.31) \qquad EX_iX_{i+j} = E[X_iX_{i+j} | X_i = 0] \Pr[X_i = 0] + E[X_iX_{i+j} | X_i > 0] \cdot \Pr[X_i > 0] \; .$$

Notice that

$$(2.32) \qquad E[X_iX_{i+j} | X_i = 0] = 0 \quad .$$

Furthermore, each positive X_j marks a regeneration point in the process. Thus if $X_i > 0$, X_i and X_{i+j} will be independent for $j = 1,2,3,\ldots$. Consequently,

$$(2.33) \qquad E[X_i X_{i+j} | X_i > 0] = E[X_i | X_i > 0]\, E[X_{i+j} | X_i > 0] \quad .$$

Using (2.32) and (2.33) , (2.31) can be rewritten as

$$(2.34) \qquad EX_i X_{i+j} = E[X_i | X_i > 0]\, Pr[X_i > 0]\, E[X_{i+j} | X_i > 0]$$

$$= \mu_r \, E[X_{i+j} | X_i > 0] \quad .$$

Therefore,

$$(2.35) \qquad \gamma_r(j) = \mu_r \{E[X_{i+j} | X_i > 0] - \mu_r\} \quad .$$

To evaluate $E[X_{i+j} | X_i > 0]$, we need an expression for the conditional expectation of X_i given that n periods have elapsed since an order was last placed. Using the conditional probabilities given in (2.13) and (2.14) , we have

$$(2.36) \qquad E[X_i | n = j] = \sum_{m=0}^{\infty} (D+m+1)\, Pr[X_i = D+m+1 | n = j]$$

$$= \left[\sum_{y=0}^{D} \phi^{j-1}(D-y) \sum_{m=0}^{\infty} (D+m+1)\phi(y+m+1) \right] / \Phi^{j-1}(D) \quad .$$

Observe that

$$(2.37) \qquad \sum_{m=0}^{\infty} (D+m+1)\phi(y+m+1) = \mu_s + D - y + \sum_{k=0}^{y} (y-D-k)\phi(k) \quad .$$

Rearranging terms yields

$$(2.38)\qquad \sum_{y=0}^{\infty} \phi^{j-1}(D-y) \sum_{k=0}^{y} (y-D-k)\phi(k) = -\sum_{y=0}^{D} (D-y) \sum_{k=0}^{D-y} \phi(k)\phi^{j-1}(D-y-k) \quad .$$

Note that the right-hand side of (2.38) is equivalent to

$$(2.39)\qquad -\sum_{k=0}^{D} k\phi^{j}(k) \quad .$$

Thus we have

$$(2.40)\qquad E[X_i|n = j] = \mu_r + \left[\sum_{k=0}^{D} k\left(\phi^{j-1}(k) - \phi^{j}(k)\right)\right]/\Phi^{j-1}(D) \quad .$$

For $j = 1$

$$(2.41)\qquad E[X_{i+1}|X_i > 0] = E[X_{i+1}|n = 1] = \mu_r - \sum_{k=0}^{D} k\phi(k) \quad .$$

For $j = 2$

$$(2.42)\qquad E[X_{i+2}|X_i > 0] = E[X_{i+2}|X_i > 0 \text{ and } X_{i+1} = 0] \Pr[X_{i+1} = 0|X_i > 0]$$

$$+ E[X_{i+2}|X_i > 0 \text{ and } X_{i+1} > 0] \Pr[X_{i+1} > 0|X_i > 0] \quad ,$$

or equivalently

$$(2.43)\qquad E[X_{i+2}|X_i > 0] = E[X_{i+2}|n = 2] \Pr[X_{i+1} = 0|n = 1]$$

$$+ E[X_{i+2}|n = 1] \Pr[X_{i+1} > 0|n = 1] \quad .$$

Employing expressions (2.13) , (2.40) , and (2.41) we obtain

$$(2.44)\qquad E[X_{i+2}|X_i > 0] = \mu_r + \Phi(D)\sum_{k=0}^{D} k\phi(k) - \sum_{k=0}^{D} k\phi^2(k)$$

$$= \mu_r - \sum_{k=0}^{D} k\phi^2(k) + \Phi(D)\{\mu_r - E[X_{i+1}|X_i > 0]\} \; .$$

Continuing in this fashion, we have the following recursive formula for $E[X_{i+j}|X_i > 0]$: for $j = 1,2,3,\ldots$,

$$(2.45)\qquad E[X_{i+j}|X_i > 0] = \begin{cases} \mu_r - \sum_{k=0}^{D} k\phi(k) & j = 1 \\ \mu_r - \sum_{k=0}^{D} k\phi^j(k) + \sum_{\ell=1}^{j-1} \Phi^{j-\ell}(D) \cdot \{\mu_r - E[X_{i+\ell}|X_i > 0]\} & j > 1 \; . \end{cases}$$

By employing (2.35) we have

$$(2.46)\qquad \gamma_r(j) = \begin{cases} -\mu_r \sum_{k=0}^{D} k\phi(k) & j = 1 \\ -\mu_r \sum_{k=0}^{D} k\phi^j(k) - \sum_{\ell=1}^{j-1} \Phi^{j-\ell}(D)\gamma_r(\ell) & j > 1 \; . \end{cases}$$

If we let $\rho_r(j)$ denote the autocorrelation at lag j of the replenishment process $\{X_i;\ i \geq 1\}$, then substitution of

$$(2.47)\qquad \rho_r(j) = \gamma_r(j)/\sigma_r^2$$

into (2.46) yields part iv of the lemma, and the proof is complete.

Notice that the first-order autocorrelation $\rho_r(1)$ is negative and becomes more negative as D increases. Both of these observations are not unexpected considering the nature of (s,S) policies. A replenishment order in one period is followed by several periods with low probability of orders being placed, and as D becomes larger, replenishment orders become less likely.

2.2 Properties of the Warehouse Demand Process

In the previous subsection we developed expressions for the mean μ_r and variance σ_r^2 of the stationary replenishment process and for the correlation between replenishment orders separated by j periods, $\rho_r(j)$. The properties of our wholesale warehouse inventory model, described in Section 1 , enable us to derive simple expressions for the mean, variance and autocorrelation function of the warehouse demand process in terms of their counterparts for the single-store replenishment processes.

Let $\mu_{r,k}$, $\sigma_{r,k}^2$, and $\gamma_{r,k}(\cdot)$ be, respectively, the mean, variance and autocovariance function of the replenishment process at store k . Let μ_w , σ_w^2 , and $\rho_w(\cdot)$ be, respectively, the mean, variance, and autocorrelation function of demand process at the warehouse. Several important properties of the warehouse demand process are given in the following lemma.

Lemma 2.2:

A. If Z_i represents the demand at the warehouse in period i,

then

i) $\mu_W = \sum_{k=1}^{N} \mu_{r,k}$

ii) $\sigma_W^2 = \sum_{k=1}^{N} \sigma_{r,k}^2$

iii) $\rho_W(j) = \sum_{k=1}^{N} \gamma_{r,k}(j) / \sum_{k=1}^{N} \sigma_{r,k}^2$ for $j = 1,2,3,\ldots$;

B. Furthermore, if each store has the same demand distribution and replenishment policy, then

iv) $\mu_W = N\mu_r$

v) $\sigma_W^2 = N\sigma_r^2$

vi) $\rho_W(j) = \rho_r(j)$, for $j = 1,2,3,\ldots$.

Proof:

Let Z_i represent the demand at the warehouse in period i. Then

$$Z_i = \sum_{k=1}^{N} X_i^k \quad , \tag{2.48}$$

where X_i^k is the order quantity received from store k in period i. Thus,

$$EZ_i = E\left[\sum_{k=1}^{N} X_i^k\right] = \sum_{k=1}^{N} EX_i^k \quad , \tag{2.49}$$

and part i is established. Since the stores operate independently of each other we have

(2.50) $$\text{Var } Z_i = \text{Var}\left[\sum_{k=1}^{N} X_i^k\right] = \sum_{k=1}^{N} \text{Var } X_i^k \quad ,$$

and part ii is established.

Let $\gamma_w(j)$ denote the autocovariance at lag j of the warehouse demand process, that is, $\gamma_w(j)$ is defined as

(2.51) $$\gamma_w(j) \equiv \text{Cov}(Z_i, Z_{i+j}) = EZ_iZ_{i+j} - \mu_w^2 \quad .$$

From (2.48) and (2.49) we have

(2.52) $$\gamma_w(j) = E\left[\sum_{k=1}^{N} X_i^k \sum_{\ell=1}^{N} X_{i+j}^{\ell}\right] - \left[\sum_{k=1}^{N} \mu_{r,k}\right]^2$$

$$= \sum_{k=1}^{N} E(X_i^k X_{i+j}^k) + \sum_{\substack{k=1 \\ k \neq \ell}}^{N} \sum_{\ell=1}^{N} E(X_i^k X_{i+j}^{\ell}) - \sum_{k=1}^{N} \mu_{r,k}^2$$

$$- \sum_{\substack{k=1 \\ k \neq \ell}}^{N} \sum_{\ell=1}^{N} \mu_{r,k}\mu_{r,\ell} \quad .$$

The independent operation of the stores allows us to rewrite (2.52) as

(2.53) $$\gamma_w(j) = \sum_{k=1}^{N} [E(X_i^k X_{i+j}^k) - \mu_{r,k}^2] = \sum_{k=1}^{N} \gamma_{r,k}(j) \quad .$$

Thus we have part iii of the lemma.

Note that if each store has the same demand distribution and replenishment policy, then the replenishment processes will be identically distributed and part B of the lemma follows directly.

3. EVALUATING THE PERFORMANCE OF WAREHOUSE INVENTORY POLICIES

In this section we discuss several cost-effective policy rules for the wholesale warehouse inventory model and describe the methods used to evaluate their performance.

3.1 Warehouse Policy Rules

Table 3.1 lists several cost-effective replenishment policies for a wholesale warehouse environment. Brief comments about the nature of the policy rules and the sections in which they are studied are also given.

Recall that our criterion for optimal warehouse inventory control is minimization of the undiscounted expected cost per period over an infinite horizon. The complexity of the warehouse demand process makes the computation of that policy which meets this criterion prohibitive. Even the form of the optimal policy is very difficult to characterize. The optimal policy will not be of a stationary (s,S) form. Nevertheless, we have confined our attention to the (s,S) form because of its popular use.

Even if we restrict the cost minimization search to only those policies of a stationary (s,S) form, the computational difficulties remain. Hence, we investigate approximately optimal (s,S) policy rules. Any such investigation of approximately optimal (s,S) policy rules, however, will be fruitless unless there is a benchmark by which we can evaluate their performance. Ideally, this benchmark is provided by the operating characteristic values of an optimal stationary (s,S) policy.

Table 3.1

Replenishment Policies For A Wholesale Warehouse Environment

POLICY RULE	COMMENTS	SECTION
OPTIMAL	Complex to characterize (not of a stationary (s,S) form).	--
MINIMUM COST STATIONARY (s,S)	Complex to compute.	--
VERY BEST (s,S)	Minimum-cost stationary (s,S) policy for a long finite history of demand.	--
BEST (s,S)	Fibonacci search for very best (s,S) policy.	4,5
I.I.D. OPTIMAL	Optimal stationary (s,S) policy for an i.i.d. demand process following a negative binomial distribution having the same mean and variance as the warehouse demands.	4
APPROXIMATELY OPTIMAL	Power Approximation corrected for correlation.	5

Unfortunately, these operating characteristic values also are prohibitively difficult to compute.

To circumvent all these difficulties we estimate an optimal stationary (s,S) policy by simulation. We seek the stationary (s,S) policy that minimizes the total cost per period for a long history of generated warehouse demands; we refer to this policy as the "very best" (s,S) policy. The software package used to find very best (s,S) policies was developed by Kaufman (1976) . The methodology of his program is outlined in the following paragraphs.

A sequence of warehouse demands is generated. A sequence of warehouse (s,S) policies is empirically tested through simulation on the generated demand sequence, and a policy with the smallest total cost per period is selected as the very best (s,S) policy.

More specifically, a sequence of values for D is examined. For each D , a corresponding value for S is selected that minimizes the total cost per period given the generated demand sequence. The sequence of D values is determined by a Fibonacci search over a range of possible values for D [Wagner (1968)] .

This technique would guarantee finding the very best (s,S) policy for the generated demand sequence if the total cost per period were convex in D ; but such convexity is generally not present [Wagner, O'Hagan, and Lundh (1965)] . Thus, the policy found may not be the very best (s,S) policy for the generated demand sequence. We refer to the policy resulting from the search technique described above as the "best" (s,S) policy.

Once a best (s,S) policy has been found, we operate the policy on the same generated demand history and collect key operating characteristic values. Since the computed operating characteristic values are statistical estimates, confidence intervals are also computed. Kaufman's program also has an option that permits the computation of operating characteristic values and confidence intervals for an arbitrary (s,S) warehouse policy. By using this option we can evaluate the performance of other (s,S) policies by comparing their operating characteristic values against those of the best (s,S) policy on an identical warehouse demand sequence.

Although the simulation program discussed above computes nearly optimal (s,S) policies, its implementation in an actual wholesale warehouse inventory system is impractical. It requires the knowledge of the demand distribution and policy parameters at each store, information usually unavailable to the warehouse manager. In addition, for a large multi-item system this method could be very expensive. For these reasons we seek policy rules that require only limited warehouse demand information.

The first rule of this type that we consider is one which ignores the sporadic and correlated nature of the warehouse demand process. This policy rule would be optimal at a facility having the same cost structure as the warehouse, but with an independent and identically distributed demand process following a negative binomial distribution having the same mean and variance as the warehouse demands. We refer to this policy rule as the I.I.D. optimal policy. Our main intent with investigating this policy rule is to see if one can afford to

ignore the sporadic and correlated nature of the warehouse demand process. An evaluation of the performance of this policy rule is presented in Section 4.

The final policy rule we consider is the Correlation-Adjusted Power Approximation. This approximately optimal policy rule, which is presented in Section 1, takes into consideration the correlated nature of the warehouse demand process. An evaluation of its performance is given in Section 5.

3.2 Experimental Design

In order to empirically evaluate the performance of alternative (s,S) policies for the wholesale warehouse inventory model, we need to establish an experimental design. This design requires the specification both of warehouse parameters and of store parameters.

3.2.1 Warehouse Parameters

We consider a full-factorial 72-item warehouse inventory system. The parameter values chosen are given in Table 3.2 . They were chosen to be consistent with those used in previous studies by Ehrhardt (1976), Kaufman (1977), and MacCormick(1974) . The four values for mean warehouse demand are 4, 8, 12, and 16 . The variance-to-mean ratio of the warehouse demand process is 9 . Three values, $L = 0, 2,$ and 4 , are assigned to leadtime. Since the cost function is linear in the parameters K , h , and p , the value of unit holding cost is set at unity. Essential changes in costs arise only with changes in the ratios K/h and p/h . The penalty cost values are $p = 4, 9,$ and 99 . The setup cost values are $K = 32$ and 64 .

Table 3.2

Warehouse Parameters

PARAMETER	PARAMETER SETTINGS	NUMBER OF SETTINGS
Mean Demand, μ_W	4, 8, 12, 16	4
Demand Variance/ Mean, σ_W^2/μ_W	9	1
Delivery Leadtime, L	0, 2, 4	3
Unit Holding Cost, h	1	1
Unit Penalty Cost, p	4, 9, 99	3
Ordering Setup Cost, K	32, 64	2

3.2.2 Store Parameters

To achieve the warehouse demand parameters given in Table 3.1 , we must appropriately specify the number of stores and the demand distribution and policy parameters at each of the stores. In addition, specification of the store parameters will induce a warehouse autocorrelation function. To aid the subsequent analysis, we specify the store parameters so that the autocorrelation function of the warehouse demand process is the same for each of the 72-items in the system.

To start, we consider a simplified environment, specifically, one in which each store employs the same (s,S) policy and has the same demand distribution. We assume that each store's demand distribution is a negative binomial distribution, with parameters r and p, where $r > 0$ and $0 < p < 1$:

$$(3.1) \qquad \phi(x) = (1-p)^r p^x \Gamma(r+x)/[\Gamma(x+1)\Gamma(r)] \quad \text{for} \quad x = 0,1,2,\ldots ,$$

yielding a mean

$$(3.2) \qquad \mu_s = rp/(1-p) \quad ,$$

a variance

$$(3.3) \qquad \sigma_s^2 = rp/(1-p)^2 \quad ,$$

and a variance-to-mean ratio

$$(3.4) \qquad \sigma_s^2/\mu_s = (1-p)^{-1} \quad .$$

It is easy to see that the mean and variance completely characterize this distribution.

For given values of N and D we can determine values for μ_s and σ_s^2 that will induce desired values of μ_w and σ_w^2. From Lemma 2.1 and Lemma 2.2 we have

$$(3.5) \qquad \mu_s = \mu_r = \mu_w/N \quad .$$

The problem of finding the appropriate value for σ_s^2, however, is much more difficult. In fact, it may not exist at all if D is too

large. To illustrate, using Lemma 2.1 and Lemma 2.2, we can write

$$\sigma_s^2 = (\sigma_w^2/N) - [2\mu_w \sum_{k=0}^{D} km(k)]/[N(1 + M(D))] \quad . \tag{3.6}$$

It can be shown that σ_s^2 is decreasing in D. For the negative binomial distribution to be well-defined, we must have $\sigma_s^2 > \mu_s$. Upon examination of (3.6) we see that it is entirely possible that a given value of D can be too large for this inequality to hold.

By the definitions given in (2.3) and (2.4), the renewal functions $m(\cdot)$ and $M(\cdot)$ depend on the probability mass function $\phi(\cdot)$, or equivalently, for the negative binomial distribution on μ_s and σ_s^2. Thus the direct solution of (3.6) is impossible. Nonetheless, we do know that if a solution does exist it must be contained in the interval $(\mu_s, \sigma_w^2/N)$, and numerical search techniques can be employed to determine an approximate solution if it exists.

Since we have postulated that each store employs the same (s,S) policy and has the same demand distribution, we have from Lemma 2.2 that

$$\rho_w(j) = \rho_r(j) \qquad \text{for } j = 1,2,3,\ldots \quad . \tag{3.7}$$

Thus $\rho_w(j)$ is independent of N for all lags j. From Lemma 2.1 we observe that $\rho_w(j)$ is, therefore, a function of μ_s, σ_s^2, and D. Thus by holding μ_s, σ_s^2, and D fixed, we can change the values of μ_w and σ_w^2 proportionally and, at the same time, keep $\rho_w(j)$ constant for all j simply by changing N in the appropriate manner. For example, to double the values of μ_w and σ_w^2 and keep

$\rho_w(j)$ fixed for all j , we simply double the number of stores N .

In this investigation we consider two warehouse demand environments. The store parameters corresponding to these demand environments are given in Table 3.3 . Both demand environments assume a value of $D = 8$ and a negative binomial demand distribution at each store. The demand environments are characterized by the number of stores in each, with one having four times as many stores as the other. In the few-stores environment each store has a mean of 4 , a variance of 12.80 , and a replenishment variance of 36 , while for the many-stores environment, the values are, respectively, 1 , 1.70 , and 9 . Thus, warehouse mean demand values of 4, 8, 12, and 16 correspond to $N = 1,2,3,$ and 4 for the few-stores environment and correspond to $N = 4,8,12,$ and 16 in the many-stores environment.

Table 3.3

Store Parameters

($D = 8$, negative binomial demand distribution at each store)

DEMAND ENVIRONMENT	N	μ_s	σ_s^2	σ_r^2
Few Stores	$\mu_w/4$	4	12.80	36
Many Stores	μ_w	1	1.70	9

The warehouse autocorrelation functions for both demand environments are listed in Table 3.4 up to lag four. For the few-stores environment the autocorrelation function has a value of -0.30 at lag one and rapidly approaches zero for lags greater than one. For the many-stores environment the autocorrelation function has a value of -0.11 at lag one and slowly goes to zero as the lag number increases. The difference between the autocorrelation functions arises from the different values for the mean and variance of the store's demand distribution.

Table 3.4
Warehouse Autocorrelation Functions

DEMAND ENVIRONMENT	$\rho_W(1)$	$\rho_W(2)$	$\rho_W(3)$	$\rho_W(4)$
Few Stores	-0.30	-0.05	+0.03	+0.01
Many Stores	-0.11	-0.10	-0.09	-0.07

Figures 3.1 and 3.2 clearly demonstrate the sporadic nature of the warehouse probability mass functions for the few-stores environment. Note for N = 1 the high probability of zero demand and the monotonically decreasing curve for demand values beyond D+1 . When

Figure 3.1

Probability Mass Functions for the Few-Stores Environment

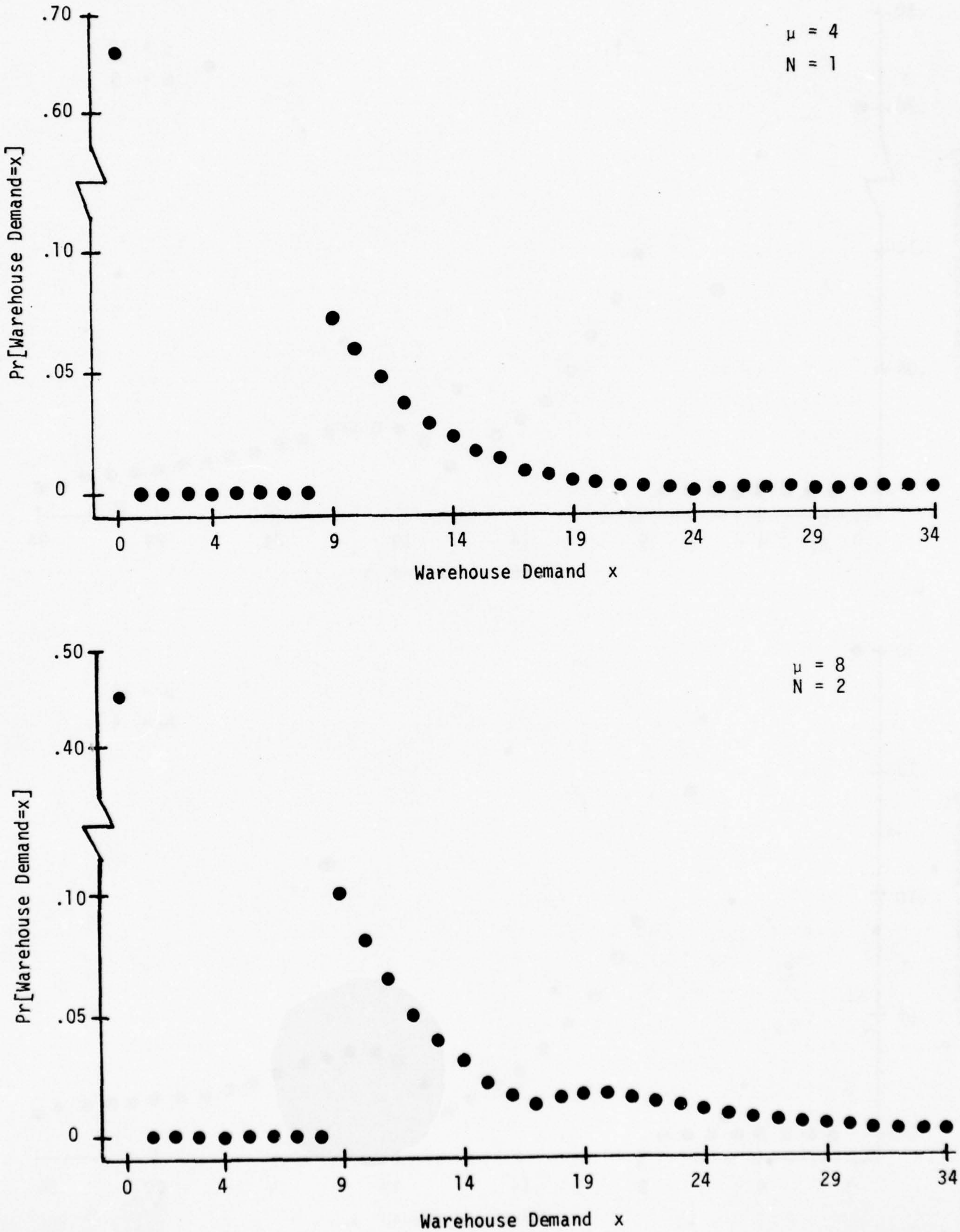
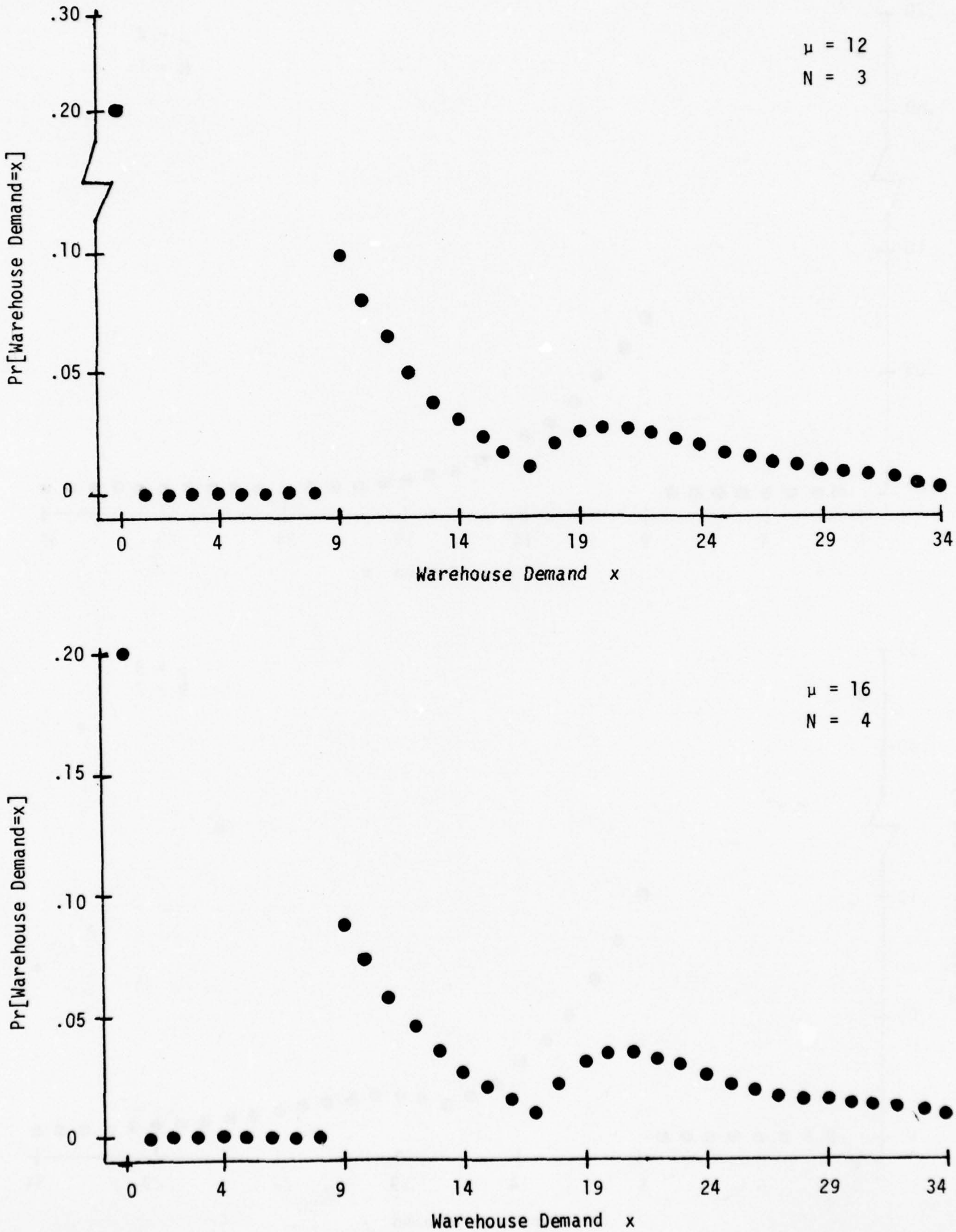


Figure 3.2

Probability Mass Functions for the Few-Stores Environment



$N = 2$ notice a slight increase in the probability mass begins at $2(D+1)$. This is the effect of the second store. For $N = 3$ and $N = 4$ we still observe this bulge which begins at $2(D+1)$; however, it becomes more pronounced when $N = 4$. The effect of more stores is to shift more mass into the tail of the distribution.

3.2.3 The Assumption of Identical Stores

In Subsection 3.2.2 we presented an experimental design in which each store employs the same (s,S) policy and has the same demand distribution. One may suspect that the warehouse demand process would be significantly affected by this assumption of identical stores. We have, therefore, computed correlation coefficients and the probability of zero demand for the warehouse demand process under several designs with non-identical stores.

Let D_i and μ_i denote, respectively, the value of D and the mean of the demand distribution at store i . The store parameter values corresponding to several designs with non-identical stores are given in Table 3.5 . For each design the warehouse demand process has a mean of 8 and a variance-to-mean ratio of 11 . In addition, the variance-to-mean ratio of each store's replenishment process is also set at 11 . Note that these variance-to-mean ratios have been increased from the previous values of 9 ; this was done in part to allow for higher D values. For each demand environment there are 3 designs. The first design has identical stores, the second has identical D values but two different mean demand values, and the third has identical mean values but two different D values.

Table 3.5

Several Designs With Non-Identical Stores

$(\mu_w = 8\ ;\ \sigma_w^2/\mu_w = \sigma_r^2/\mu_r = 11)$

DEMAND ENVIRONMENT	DESIGN	VALUES OF D	VALUES OF μ_s
Few Stores	I	$D_1 = D_2 = 8$	$\mu_1 = \mu_2 = 4$
	II	$D_1 = D_2 = 8$	$\mu_1 = 2;\ \mu_2 = 6$
	III	$D_1 = 6;\ D_2 = 10$	$\mu_1 = \mu_2 = 4$
Many Stores	IV	$D_1 = \ldots = D_8 = 8$	$\mu_1 = \ldots = \mu_8 = 1$
	V	$D_1 = \ldots = D_8 = 8$	$\mu_1 = \ldots = \mu_4 = 0.5;$ $\mu_5 = \ldots = \mu_8 = 1.5$
	VI	$D_1 = \ldots = D_4 = 6;$ $D_5 = \ldots = D_8 = 10$	$\mu_1 = \ldots = \mu_8 = 1$

Table 3.6 gives the probability that warehouse demand in period i, Z_i, is zero along with several autocorrelation coefficients for each of the designs given in Table 3.5 .

Comparing the autocorrelation coefficients of designs I and IV with those given in Table 3.4 , we see that increasing σ_w^2/μ_w from 9 to 11 results in less correlation between warehouse demands but similar behavior of the autocorrelation function. By comparing design I with designs II and III and design IV with designs V and VI , we see

that no significant changes have occurred either in the probability of zero demand or in the autocorrelation coefficients. These results suggest that the assumption of identical stores is not a critical factor in influencing the nature of the warehouse demand process.

Table 3.6

Probability of Zero Demand and Autocorrelation Coefficients for the Warehouse Demand Process Under Several Non-Identical Store Designs

DEMAND ENVIRONMENT	DESIGN	$Pr[Z_i = 0]$	$\rho_w(1)$	$\rho_w(2)$	$\rho_w(3)$	$\rho_w(4)$
Few Stores	I	0.488	-0.20	-0.06	0.00	+0.01
	II	0.490	-0.20	-0.03	0.00	0.00
	III	0.480	-0.21	-0.07	+0.01	+0.01
Many Stores	IV	0.473	-0.07	-0.06	-0.05	-0.05
	V	0.475	-0.09	-0.07	-0.05	-0.03
	VI	0.465	-0.07	-0.06	-0.06	-0.05

3.3 Item Operating Characteristics

In Sections 4 and 5 the evaluation of performance of alternative (s,S) policy rules, under the experimental design described above, is made by comparing their operating characteristic values with those

obtained using best (s,S) policies in the same environment. Operating characteristic values for each item in the 72-item system are collected for 5000 periods. The operating characteristics we consider are the expected values per period of period-end inventory, backlog quantity, frequency of period-end backlogs, replenishment quentity, frequency of replenishment and the total cost incurred.

3.4 The Multi-Item System

Scientific techniques for inventory control are generally applied to systems of many items. Hence in Sections 4 and 5 we also combine the results from two 72-item systems into two multi-item systems. Since management generally assesses the performance of control techniques by observing indices that are aggregate operating characteristics [Wagner (1962)] , certain aggregate characteristics have been computed.

The operating characteristics of the multi-item system have been measured by aggregating the sample values of the corresponding characteristics for each item in the system. The aggregate of average total cost per period is computed as the arithmetic sum of the corresponding costs for each item. The components of total cost for inventory storage, backlog penalty, and ordering replenishments are similarly computed. The aggregate backlog and replenishment frequencies are arithmetic averages of the corresponding frequencies observed for each item in the system. Since the unit inventory holding cost for all items is unity, the average number of units in inventory at period-end is numerically identical to the aggregate holding cost per period. Finally, a weighted sum of the average quantity backlogged per period to a weighted sum of

the (exact) mean values of demand. The weights used in both the numerator and denominator of the ratio are the unit cost of backlogging demand for the respective item.

4. EVALUATION OF A POLICY RULE THAT IGNORES SPORADIC AND CORRELATED DEMAND PROPERTIES

In this section we evaluate the performance of the I.I.D. optimal policy introduced in Table 3.1 . Recall that under this policy rule we compute optimal policies for items having the warehouse cost structure, but with an independent and identically distributed demand process following a negative binomial distribution having the same mean and variance as the warehouse demand process. The policies are computed by the algorithm of Veinott and Wagner (1965) as programmed by Kaufman (1976) . Essentially, this policy rule ignores the sporadic and correlated nature of the warehouse demand environment. The purpose of testing the I.I.D. optimal policy rule in the warehouse demand environment is to see if one can afford to ignore these properties. It is particularly important that this issue be studied, since practitioners often make this kind of simplifying assumption.

We find that the use of the I.I.D. optimal policy rule in our warehouse demand environment results in significant degradation in performance. Specifically, the total cost performance of the I.I.D. optimal policies is about 8% above that of the best (s,S) policies, as defined in Table 3.1 . This degradation is the result of holding too much inventory.

4.1 The Performance of I.I.D. Optimal Policies

Table 4.1 summarizes the performance of the I.I.D. optimal policies in both warehouse demand environments. The components of aggregate average total cost per period are listed and are compared with the

Table 4.1

Summary of the Performance of the I.I.D. Optimal Policies for Two

72-Item Warehouse Inventory Systems

COST COMPONENT	FEW-STORES ENVIRONMENT		MANY-STORES ENVIRONMENT	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	2288 (68.3)	541 [31.0]	2297 (67.2)	462 [25.1]
BACKLOG	311 (9.3)	-155 [-33.4]	369 (10.8)	-112 [-23.3]
REPLENISHMENT	752 (22.5)	-116 [-13.3]	752 (22.0)	-122 [-14.0]
TOTAL	3351 (100.0)	270 [8.8]	3417 (100.0)	227 [7.1]

NOTE: Numbers in parentheses are percentages of total cost.
Numbers in brackets are percentage differences in cost components over best values.

corresponding cost when the systems are controlled by best (s,S) policies. Notice that the total cost performance of the I.I.D. optimal policies is 8.8% and 7.1% higher than the best values for the few-stores environment and the many-stores environment, respectively. The cause of this degradation in performance is clearly the result of holding too much inventory. Specifically, 31% more inventory is held in the few-stores environment, and 25% more in the many-stores environment. In addition, inventory costs account for approximately two-thirds of the total cost in both environments. Observe that backlog and replenishment costs are significantly lower in both environments; however, the savings over best values are not subtantial enough to offset the rise in inventory holding costs.

Insight into the cost component differences is gained by comparing the policy parameter values s and S of the best policies with those of the I.I.D. optimal policies. Table 4.2 lists policy parameter values for a subset of the 72-item few-stores environment. Note that for the best policies D = S-s is smaller for all items except one, s is smaller for all items except those with zero leadtime and small penalty cost, and S is smaller with the exception of two items. This subset is representative of all 72-items and both environments. Thus, in general, D and S tend to be smaller, and s is smaller except for those items with zero leadtime and small unit penalty costs. A complete comparison of the policy parameters is given in Technical Report 15. The smaller D values account for the larger replenishment costs for the best policies, and the smaller s values

Table 4.2

Comparison of Policy Parameter Values for a Subset of the 72-Item System Under Few-Stores Environment

INPUT PARAMETERS				BEST POLICIES			I.I.D. OPTIMAL POLICIES		
L	p	K	μ	s	S	D	s	S	D
0	9	32	4	7	22	15	3	20	17
0	9	32	16	20	44	24	16	47	31
0	9	64	4	5	22	17	1	25	24
0	9	64	16	14	60	46	13	57	44
0	99	32	4	14	29	15	19	36	17
0	99	32	16	36	59	23	41	70	29
0	99	64	4	14	29	15	17	41	24
0	99	64	16	34	69	35	38	81	43
4	9	32	4	24	39	15	27	48	21
4	9	32	16	88	118	30	95	133	38
4	9	64	4	21	48	27	24	52	28
4	9	64	16	85	127	42	90	143	53
4	99	32	4	37	52	15	52	72	20
4	99	32	16	115	137	22	136	171	35
4	99	64	4	37	52	15	50	77	27
4	99	64	16	111	152	41	132	181	49

for those items with high penalty costs result in increased aggregate backlog costs. However, the S values which are also smaller, particularly so for those items with a larger penalty cost and positive leadtime, result in considerable holding cost savings.

Detailed comparisons of the systems under the two decisions rules, expanding on Table 4.1 , are found in Tables 4.3 and 4.4 showing percentages above best values for each cost component and parameter setting using the I.I.D. optimal policies. The total cost data reveal that the I.I.D. optimal policies are close to best (s,S) policies only for those items with small penalty costs or those with zero leadtime. High penalty costs and large leadtimes are particularly troublesome, and to a somewhat lesser degree, so are low means and small setup costs. Inventory costs are considerably above best values for all parameter settings, especially for a high penalty cost or a large leadtime. Backlog costs are below best values for nearly all of the parameter settings. Note, in particular, that backlog costs are considerably less when the penalty cost is 99 or when the leadtime is 4 . Replenishment costs are below best values for all parameter settings and, similar to backlog costs, have the largest discrepancy for those items with high penalty costs or large leadtimes. Cost component differences are similar for both environments, the many-stores environment exhibiting slightly smaller differences.

The percentage apportionment of aggregate costs per period for various parameter classifications is shown in Tables 4.5 and 4.6 for best (s,S) policies and for the I.I.D. optimal (s,S) policies.

Table 4.3

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
I.I.D. OPTIMAL POLICIES													
INVENTORY	31.0	13.9	21.1	44.0	35.3	27.2	10.3	34.5	43.6	33.0	33.3	30.2	29.0
BACKLOG	-33.4	- 8.2	-23.5	-82.4	-38.0	-28.8	2.0	-40.1	-53.3	-24.3	-27.7	-43.8	-32.9
REPLENISHMENT	-13.3	- 9.6	-12.1	-17.6	-15.1	-12.1	- 7.8	-15.0	-17.3	-13.8	-19.0	- 7.3	-14.0
TOTAL	8.8	1.3	3.6	17.2	10.6	7.3	3.4	9.4	12.6	11.6	8.7	8.7	7.4

Table 4.4

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
I.I.D. OPTIMAL POLICIES													
INVENTORY	25.1	9.5	16.3	36.9	27.9	22.7	12.5	22.3	36.7	28.5	28.1	23.5	22.8
BACKLOG	-23.3	2.8	-12.7	-77.2	-26.3	-20.4	- 5.4	-21.9	-39.6	-11.6	-14.2	-37.4	-23.0
REPLENISHMENT	-14.0	-11.7	-14.8	-15.2	-17.5	-11.5	-11.5	-13.4	-17.2	-18.1	-21.9	- 4.3	-13.8
TOTAL	7.1	1.1	2.2	14.5	7.9	6.5	2.0	6.4	12.1	9.9	7.3	7.0	5.7

Inventory costs for the I.I.D. optimal policies increase more dramatically for higher penalty costs and for positive leadtimes than for best policies. Backlog costs, on the other hand, decrease far more sharply as penalty costs increase. Notice that as the leadtime increases, the backlog costs increase for the best policies and decrease for the I.I.D. optimal policies.

Tables 4.7 and 4.8 show the values of other operating characteristics of the systems under best policies and I.I.D. optimal policies. Backlog and replenishment frequency are higher for the best policies. Backlog frequency increases only slightly in leadtime for best policies and decreases sharply for I.I.D. optimal policies. Higher setup costs lead to less frequent backlogs for the best policies and more frequent backlogs for the I.I.D. optimal policies. Percentages above best values for the operating characteristics of the I.I.D. optimal policies are given in Tables 4.9 and 4.10 . The results are consistent with trends noted for the cost components.

The above performance summary indicates that the I.I.D. optimal policy rule is unsatisfactory, especially for those items with a high penalty cost or a long leadtime. In general, the I.I.D. optimal policies hold too much inventory and order too infrequently. Hence, it can be very costly to ignore the sporadic and correlated nature of the warehouse demand process. What is needed is a policy rule that can adjust itself to these demand characteristics in a simple way. In Section 5 we present a policy rule which adjusts for correlation in a demand process.

Table 4.5

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	56.7	11.9	16.5	28.2	26.5	30.2	16.3	18.9	21.4	9.4	12.8	16.3	18.3
BACKLOG	15.1	5.8	5.3	4.0	7.5	7.6	4.3	5.0	5.9	2.5	3.5	4.3	4.8
REPLENISHMENT	28.2	8.8	9.2	10.2	11.3	16.9	9.6	9.4	9.2	4.4	6.7	7.8	9.3
TOTAL	100.0	26.5	31.0	42.5	45.3	54.7	30.1	33.3	36.5	16.3	23.0	28.3	32.4
I.I.D. OPTIMAL POLICIES													
INVENTORY	68.3	12.5	18.4	37.4	32.9	35.4	16.5	23.4	28.3	11.5	15.7	19.5	21.6
BACKLOG	9.3	4.9	3.7	0.7	4.3	5.0	4.0	2.7	2.5	1.8	2.3	2.2	3.0
REPLENISHMENT	22.5	7.3	7.4	7.7	8.8	13.7	8.1	7.4	7.0	3.5	5.0	6.6	7.4
TOTAL	100.0	24.7	29.5	45.8	46.0	54.0	28.6	33.5	37.8	16.7	23.0	28.3	32.0

Table 4.6

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	57.5	12.1	16.7	28.8	27.0	30.5	15.5	20.2	21.7	9.5	12.9	16.6	18.5
BACKLOG	15.1	5.9	5.3	3.9	7.5	7.6	4.5	5.1	5.4	2.5	3.4	4.3	4.9
REPLENISHMENT	27.4	8.6	9.2	9.6	11.2	16.2	9.6	9.1	8.8	4.4	6.7	7.3	9.0
TOTAL	100.0	26.6	31.2	42.3	45.7	54.3	29.7	34.4	36.0	16.4	23.0	28.2	32.4
I.I.D. OPTIMAL POLICIES													
INVENTORY	67.2	12.4	18.1	36.7	32.3	34.9	16.3	23.1	27.8	11.4	15.5	19.2	21.2
BACKLOG	10.8	5.6	4.3	0.8	5.2	5.6	4.0	3.7	3.0	2.0	2.7	2.5	3.5
REPLENISHMENT	22.0	7.1	7.3	7.6	8.6	13.4	8.0	7.2	6.8	3.4	4.9	6.5	7.2
TOTAL	100.0	25.1	29.7	45.2	46.0	54.0	28.3	34.1	37.6	16.8	23.1	28.1	31.9

Table 4.7

Operating Characteristics of a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1747	369	508	870	816	931	503	583	660	290	395	501	563
Backlog Frequency	.097	.191	.093	.009	.098	.097	.096	.097	.099	.093	.098	.099	.099
Weighted Proportion of Demand Backlogged	.017	.186	.076	.005	.017	.018	.015	.017	.020	.029	.020	.017	.014
Replenishment Frequency	.264	.246	.258	.287	.303	.225	.260	.267	.256	.161	.253	.291	.351
I.I.D. OPTIMAL POLICIES													
Period-End Inventory	2288	419	615	1254	1103	1185	554	785	951	384	526	652	725
Backlog Frequency	.085	.179	.074	.002	.082	.087	.108	.078	.068	.090	.089	.075	.085
Weighted Proportion of Demand Backlogged	.012	.171	.058	.001	.011	.013	.015	.010	.001	.022	.014	.009	.009
Replenishment Frequency	.228	.222	.226	.235	.257	.198	.247	.224	.212	.140	.203	.268	.299

Table 4.8

Operating Characteristics of a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1835	385	532	918	861	974	494	646	694	303	413	528	589
Backlog Frequency	.097	.188	.095	.009	.097	.097	.095	.098	.099	.093	.097	.099	.100
Weighted Proportion of Demand Backlogged	.018	.195	.079	.005	.018	.018	.016	.018	.019	.029	.020	.017	.015
Replenishment Frequency	.267	.251	.268	.283	.310	.224	.283	.264	.254	.169	.258	.286	.355
I.I.D. OPTIMAL POLICIES													
Period-End Inventory	2297	423	620	1256	1103	1194	556	790	947	390	528	655	724
Backlog Frequency	.092	.191	.083	.002	.092	.092	.100	.095	.081	.099	.098	.079	.092
Weighted Proportion of Demand Backlogged	.014	.200	.069	.001	.013	.014	.015	.014	.012	.026	.017	.011	.011
Replenishment Frequency	.227	.220	.226	.236	.256	.198	.248	.224	.209	.139	.203	.268	.298

Table 4.9

Percentage Above Best (s,S) Operating Characteristics for a 72-Item Warehouse System Under I.I.D. Optimal Policy Control

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
I.I.D. OPTIMAL POLICIES													
Period-End Inventory	31.0	13.9	21.1	44.0	35.3	27.2	10.3	34.5	43.6	33.0	33.3	30.2	29.0
Backlog Frequency	-13.1	- 6.3	-20.7	-81.6	-16.4	- 9.8	11.7	-18.9	-31.6	- 3.9	- 9.2	-24.4	-14.5
Weighted Proportion of Demand Backlogged	-33.4	- 8.2	-23.5	-82.4	-38.0	-28.8	2.0	-40.1	-53.3	-24.3	-27.7	-43.8	-32.9
Replenishment Frequency	-13.8	-10.0	-12.5	-18.3	-15.1	-12.1	- 8.6	-16.0	-17.1	-13.4	-19.7	- 7.9	-14.7

Table 4.10

Percentage Above Best (s,S) Operating Characteristics for a 72-Item Warehouse System Under I.I.D. Optimal Policy Control

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
I.I.D. OPTIMAL POLICIES													
Period-End Inventory	25.1	9.5	16.3	36.9	27.9	22.7	12.5	22.3	36.7	28.5	28.1	23.5	22.8
Backlog Frequency	- 5.4	1.7	-12.6	-77.7	- 5.6	- 5.2	5.3	- 2.8	-18.3	6.5	0.5	-20.1	- 7.4
Weighted Proportion of Demand Backlogged	-23.3	2.8	-12.7	-77.2	-26.3	-20.4	- 5.4	-21.9	-39.6	-11.6	-14.2	-37.4	-23.0
Replenishment Frequency	-15.0	-12.3	-15.7	-16.8	-17.5	-11.5	-12.4	-15.1	-17.7	-17.9	-21.3	- 6.2	-16.1

5. THE POWER APPROXIMATION IN A WAREHOUSE DEMAND ENVIRONMENT

In the previous section we observed that the correlated and sporadic nature of the warehouse demand process cannot be overlooked without serious degradation in system performance. Klincewicz (1976) studied the effectiveness of the Power Approximation in a particular sporadic demand environment in which demands are independent and identically distributed. He studied demand distributions that have considerable probability of no demand in a period and also have a high variance-to-mean ratio. His results indicate that the Power Approximation performance in such an environment is close to optimal. Although more pronounced, the sporadic effect in the warehouse demand environment is similar. For this reason we have concentrated on the correlation effects of the warehouse demand process on the policy parameters.

In Subsection 5.1 we present an adaptation of the Power Approximation to a correlated demand environment. The modified algorithm requires only knowledge of the mean, variance, and autocorrelation function up to lag L. In Subsection 5.2 we evaluate the performance of the Correlation-Adjusted Power Approximation. We find its performance to be close to the performance of the best (s,S) policies. Specifically, for the few-stores and many-stores environments the policies yield costs of only 2.6% and 3.4%, respectively, above expected costs for the best policies.

5.1 Adaptation of the Power Approximation to Correlated Demands

The Power Approximation is an algorithm for computing approximately optimal values for (s,S) using only the mean and variance of demand. Ehrhardt (1976) has demonstrated that the algorithm is a highly accurate approximation to optimal (s,S) policies. For this reason the performance of Power Approximation policies in a warehouse demand environment will closely resemble that of the I.I.D. optimal policies. Nonetheless, we can modify the Power Approximation for use in a correlated demand environment.

In the Power Approximation expressions for s and D we find the term

$$\sigma^2_{L+1} = (L + 1)\sigma^2 \tag{5.1}$$

where σ^2 represents the variance of the demand process. Since the Power Approximation was derived for independent, identically distributed demands, the term σ^2_{L+1} represents the variance of demand over $L+1$ periods. We modify the Power Approximation by simply replacing (5.1) with the more general expression

$$\sigma^2_{L+1} = [(L+1) + 2 \sum_{j=1}^{L} (L+1-j)\rho(j)]\sigma^2 \quad , \tag{5.2}$$

where $\rho(j)$ is the autocorrelation of the process at lag j. We refer to the Power Approximation with (5.2) replacing (5.1) as the Correlation-Adjusted Power Approximation.

Upon examination of the autocorrelation functions for both demand

environemnts in Table 3.2 , it is clear that the correlation-adjusted variance over L+1 periods will be less than $(L+1)\sigma^2$. Table 5.1 lists this adjusted variance for several values of σ^2 and L and for both demand environments. Notice the significant reduction in the variance over L+1 periods, especially for the few-stores environment, where reductions are in the neighborhood of 40 to 50 percent.

Table 5.2 compares Power Approximation policies using (5.1) and (5.2) for a subset of the 72-item systems. The values of D are only 1 or 2 units smaller, since σ^2_{L+1} is raised to a very small power in the expression for D . However, s has decreased substantially, especially when the penalty cost is 99 . By comparing with best parameter values, we see that for most items the changes in D and s are in the appropriate direction. Note also that the policies change more dramatically for the few-stores environment.

5.2 The Performance of Correlation-Adjusted Power Approximation Policies

Table 5.3 summarizes the performance of the Correlation-Adjusted Power Approximation policies for both warehouse demand environments. The components of aggregate average total cost per period are listed and are compared with the corresponding costs when the systems are controlled by best (s,S) policies. Notice the total cost component is only 2.6% above best (s,S) policy performance for the few-stores environment, and 3.4% above for the many-stores environment. Inventory holding costs are above best values, while backlog and replenishment costs are below best values. Recall that the I.I.D. optimal policies displayed the same characteristics; however, here the differences are

Table 5.1

Comparison of Variance Over L+1 Periods for

Uncorrelated and Correlated Demands

LEADTIME L	ONE PERIOD VARIANCE σ^2	UNADJUSTED σ^2_{L+1} $\sigma^2(L+1)$	CORRELATION-ADJUSTED σ^2_{L+1}	
			FEW-STORES	MANY-STORES
2	36	108	61	85
2	72	216	121	169
2	108	324	182	253
2	144	432	242	338
4	36	180	86	107
4	72	360	172	214
4	108	540	258	321
4	144	720	344	428

Table 5.2

Comparison of Policy Parameter Values for a Subset of the 72-Item Systems

				Power Apprx. Using (5.1)		Power Apprx. Using (5.2)				Best			
						Few-Stores		Many-Stores		Few-Stores		Many-Stores	
L	P	K	μ	s	D	s	D	s	D	s	D	s	D
2	9	32	4	17	18	14	18	15	19	14	15	19	11
2	9	32	16	58	34	52	33	55	34	52	30	56	25
2	9	64	4	13	27	11	26	12	26	12	22	16	22
2	9	64	16	52	48	48	46	50	48	49	42	51	44
2	99	32	4	37	19	30	18	34	18	27	14	31	11
2	99	32	16	95	34	81	33	88	34	76	21	79	26
2	99	64	4	34	26	27	26	31	26	24	27	29	20
2	99	64	16	88	49	76	47	83	47	73	33	76	41
4	9	32	4	28	20	23	19	25	18	24	15	26	14
4	9	32	16	96	36	87	34	90	34	88	30	91	25
4	9	64	4	24	28	20	26	21	27	21	27	25	22
4	9	64	16	90	50	82	48	84	48	85	42	86	44
4	99	32	4	54	20	42	19	45	19	37	15	39	11
4	99	32	16	143	36	121	34	127	34	115	22	117	22
4	99	64	4	50	27	39	26	42	26	37	15	39	11
4	99	64	16	135	50	115	48	120	49	111	41	111	44

Table 5.3

Summary of the Performance of Correlation-Adjusted Power Approximation

Policies for Two 72-Item Warehouse Inventory Systems

COST COMPONENT	FEW-STORES ENVIRONMENT		MANY-STORES ENVIRONMENT	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	1930 (61.1)	183 [10.5]	2080 (63.1)	225 [13.3]
BACKLOG	461 (14.6)	- 5 [- 1.0]	454 (13.8)	- 27 [- 5.5]
REPLENISHMENT	768 (24.3)	-100 [-11.5]	765 (23.2)	-109 [-12.5]
TOTAL	3160 (100.0)	79 [2.6]	3299 (100.0)	109 [3.4]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

less severe. Substantial improvement has been made in the inventory cost component due to lower values of S . In particular, inventory costs for the few-stores environment are down from 31% to 10.5% above best values. Replenishment costs for both environments are, respectively, 11.5% and 12.5% below best values. Since D is virtually unchanged, these percentages are only slightly smaller than the corresponding percentages when using the I.I.D. optimal policies. Backlog costs are, respectively, 1% and 5.5% below best values for the few-stores environment and the many-stores environment, and are much closer to best values than the corresponding backlog costs using the I.I.D. optimal policies. This can be attributed to the lower values of s .

Tables 5.4 and 5.5 show percentages above best values for each cost component and parameter setting suing the Correlation-Adjusted Power Approximation policies. The total cost component reveals that, with the exception of those items with high penalty cost, the Correlation-Adjusted Power Approximation performance is close to best performance. Comparing with Tables 4.3 and 4.4 , we note that the severe degradation for large leadtimes with the I.I.D. optimal policies is not longer present. That is, our revised calculation of leadtime variance has effected a substantial improvement for those items with positive leadtime.

The percentage apportionment of aggregate costs per period for various parameter classifications is shown in Tables 5.6 and 5.7 for best (s,S) policies and for Correlation-Adjusted Power Approximation policies. Note that backlog costs increase slightly as

Table 5.4

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
INVENTORY	10.5	- 4.0	3.2	20.9	15.1	6.5	9.2	11.0	11.1	12.0	12.5	8.4	10.3
BACKLOG	- 1.0	20.3	12.6	-49.5	-14.1	11.8	7.3	- 1.8	- 6.4	1.8	4.5	- 3.2	- 4.6
REPLENISHMENT	-11.5	- 5.4	-10.0	-18.2	-13.1	-10.5	-10.8	-12.4	-11.4	-11.7	-15.8	- 8.0	-11.3
TOTAL	2.6	0.9	0.9	4.8	3.2	2.0	2.6	2.5	2.6	4.1	3.0	2.1	1.9

Table 5.5

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
<u>CORR. - ADJUSTED P.A.</u>													
INVENTORY	13.3	- 1.7	5.7	24.1	17.5	9.6	11.1	13.0	15.2	18.5	16.7	8.4	12.7
BACKLOG	- 5.5	17.4	9.1	-60.0	-14.6	3.4	2.2	- 6.0	-11.7	-13.3	- 3.1	- 0.2	- 8.1
REPLENISHMENT	-12.5	- 7.2	-13.2	-16.5	-15.6	-10.3	-14.5	-11.8	-10.9	-13.2	-18.3	- 7.9	-11.5
TOTAL	3.4	0.7	0.7	7.1	4.1	2.8	1.4	3.7	4.8	5.2	3.6	2.9	2.9

Table 5.6

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	56.7	11.9	16.5	28.2	26.5	30.2	16.3	18.9	21.4	9.4	12.8	16.3	18.3
BACKLOG	15.1	5.8	5.3	4.0	7.5	7.6	4.3	5.0	5.9	2.5	3.5	4.3	4.8
REPLENISHMENT	28.2	8.8	9.2	10.2	11.3	16.9	9.5	9.4	9.2	4.4	6.7	7.8	9.3
TOTAL	100.0	26.5	31.0	42.5	45.3	54.7	30.1	33.3	36.5	16.3	23.0	28.3	32.4
CORR. - ADJUSTED P.A.													
INVENTORY	61.1	11.2	16.6	33.3	29.7	31.4	17.4	20.5	23.2	10.3	14.1	17.2	19.6
BACKLOG	14.6	6.8	5.8	2.0	6.3	8.3	4.5	4.7	5.4	2.5	3.5	4.1	4.5
REPLENISHMENT	24.3	8.1	7.8	8.2	9.6	14.7	8.3	8.1	7.9	3.7	5.5	6.9	8.1
TOTAL	100.0	26.1	30.5	43.4	45.6	54.4	30.1	33.3	36.5	16.5	23.1	28.2	32.2

Table 5.7

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
<u>BEST POLICIES</u>													
INVENTORY	57.5	12.1	16.7	28.8	27.0	30.5	15.5	20.2	21.7	9.5	12.9	16.6	18.5
BACKLOG	15.1	5.9	5.3	3.9	7.5	7.6	4.5	5.1	5.4	2.5	3.4	4.3	4.9
REPLENISHMENT	27.4	8.6	9.2	9.6	11.2	16.2	9.6	9.1	8.8	4.4	6.7	7.3	9.0
TOTAL	100.0	26.6	31.2	42.3	45.7	54.3	29.7	34.4	36.0	16.4	23.0	28.2	32.4
<u>CORR. - ADJUSTED P.A.</u>													
INVENTORY	63.1	11.5	17.0	34.5	30.7	32.4	16.7	22.1	24.3	10.9	14.6	17.4	20.2
BACKLOG	13.8	6.7	5.6	1.5	6.2	7.6	4.5	4.7	4.6	2.1	3.1	4.2	4.4
REPLENISHMENT	23.2	7.7	7.7	7.8	9.2	14.0	8.0	7.7	7.4	3.7	5.3	6.5	7.7
TOTAL	100.0	25.9	30.4	43.8	46.0	54.0	29.1	34.5	36.4	16.7	23.1	28.0	32.2

leadtime increases, which is now in agreement with best policy performance.

Tables 5.8 and 5.9 show the values of other operating characteristics of the systems under best policies and Correlation-Adjusted Power Approximation policies. Note that for the Correlation-Adjusted Power Approximation, backlog frequency is slightly dependent on setup costs, whereas best policies exhibit no such dependence. Percentages above best values for the operating characteristics of the Correlation-Adjusted Power Approximation policies are given in Tables 5.10 and 5.11 . Note backlog frequencies are above best values except for high penalty cost items, in which backlog frequency is substantially below the best value.

Table 5.8

Operating Characteristics of a 72-Item Warehouse Inventory System

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS: PENALTY 4	PENALTY 9	PENALTY 99	SETUP 32	SETUP 64	LEADTIME 0	LEADTIME 2	LEADTIME 4	MEAN 4	MEAN 8	MEAN 12	MEAN 16
BEST POLICIES													
Period-End Inventory	1747	369	508	870	816	931	503	583	660	290	395	501	563
Backlog Frequency	.097	.191	.093	.009	.098	.097	.096	.097	.099	.093	.098	.099	.099
Weighted Proportion of Demand Backlogged	.017	.186	.076	.005	.017	.018	.015	.017	.020	.029	.020	.017	.014
Replenishment Frequency	.254	.246	.258	.287	.303	.225	.260	.267	.256	.161	.253	.291	.351
CORR. - ADJUSTED P.A.													
Period-End Inventory	1930	353	525	1052	938	992	548	647	734	324	444	542	620
Backlog Frequency	.109	.221	.100	.004	.100	.118	.112	.108	.105	.109	.112	.106	.108
Weighted Proportion of Demand Backlogged	.017	.224	.085	.003	.015	.020	.016	.017	.019	.029	.021	.016	.013
Replenishment Frequency	.232	.233	.232	.233	.263	.202	.239	.231	.227	.144	.211	.266	.308

Table 5.9

Operating Characteristics of a 72-Item Warehouse Inventory System

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1835	335	532	918	861	974	494	646	694	303	413	528	589
Backlog Frequency	.097	.188	.095	.009	.097	.097	.095	.098	.099	.093	.097	.099	.100
Weighted Proportion of Demand Backlogged	.018	.195	.079	.005	.018	.018	.016	.018	.019	.029	.020	.017	.015
Replenishment Frequency	.267	.251	.268	.283	.310	.224	.283	.264	.254	.169	.258	.286	.355
CORR. - ADJUSTED P.A.													
Period-End Inventory	2080	379	562	1138	1013	1067	549	730	801	360	483	574	666
Backlog Frequency	.104	.212	.096	.003	.099	.108	.103	.105	.104	.098	.105	.108	.103
Weighted Proportion of Demand Backlogged	.017	.229	.086	.002	.015	.019	.017	.017	.017	.026	.019	.017	.013
Replenishment Frequency	.231	.232	.230	.232	.262	.201	.239	.229	.226	.149	.213	.258	.306

Table 5.10

Percentage Above Best (s,S) Operating Characteristics for a 72-Item Warehouse System Under Correlation Adjusted Power Approximation Policy Control

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
Period-End Inventory	10.5	- 4.0	3.2	20.9	15.1	6.5	9.2	11.0	11.1	12.0	12.5	8.4	10.3
Backlog Frequency	11.5	15.9	8.5	-51.5	1.7	21.3	16.3	12.0	6.3	16.3	14.5	6.9	8.4
Weighted Proportion of Demand Backlogged	- 1.0	20.3	12.6	-49.5	-14.1	11.8	7.3	- 1.8	- 6.4	1.8	4.5	- 3.2	- 4.6
Replenishment Frequency	-12.0	- 5.6	-10.3	-19.0	-13.1	-10.5	-11.6	-13.2	-11.0	-10.4	-16.6	- 8.5	-12.2

Table 5.11

Percentage Above Best (s,S) Operating Characteristics for a 72-Item Warehouse System Under Correlation Adjusted Power Approximation Policy Control

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
Period-End Inventory	13.3	- 1.7	5.7	24.1	17.5	9.6	11.1	13.0	15.2	18.5	16.7	8.4	12.7
Backlog Frequency	6.6	12.5	1.3	-62.4	1.7	11.5	7.8	7.3	4.7	6.0	8.1	9.0	3.2
Weighted Proportion of Demand Backlogged	- 5.5	17.4	9.1	-60.0	-14.6	3.4	2.2	- 6.0	-11.7	-13.3	- 3.1	- 0.2	- 8.1
Replenishment Frequency	-13.4	- 7.4	-14.0	-18.1	-15.6	-10.3	-15.5	-13.4	-11.0	-12.0	-17.5	- 9.8	-14.0

6. CONCLUSIONS

In this section we summarize the principal results of this report and outline topics to be investigated further.

In Section 2 we presented a detailed analysis of the demand process for our wholesale warehouse inventory model. We were able to derive useful expressions for its mean, variance, and autocorrelation function in terms of the policy parameters and demand distribution at each store.

We considered several (s,S) policy rules for the wholesale warehouse. In Section 4 we examined a policy rule which ignores both the sporadic and the correlated nature of the warehouse demand process. This policy rule is *optimal if* demands are independent and identically distributed negative binomial random variables with a mean and variance equal to that of the warehouse demand process. We found the performance of this policy rule to be unsatisfactory in several warehouse demand environments; particularly, when the penalty cost is high or the leadtime is large. In general, the policy rule holds too much inventory and orders too infrequently. In Section 5 we examined a policy rule which takes into account the correlated nature of the demand process. This policy rule is a simple modification of the Ehrhardt Power Approximation. The only demand information required is the mean and variance of demand and the variance of demand over one leadtime, σ^2_{L+1} . We found that this policy performed quite well in the same warehouse demand environments.

When using the Correlation-Adjusted Power Approximation in this

study, we assumed that μ, σ^2, and σ^2_{L+1} are known. How this policy rule performs when these demand parameters must be estimated from a limited demand history is an important issue still to be investigated. The main difficulty foreseen is the accurate estimation of σ^2_{L+1}. One possible method is to estimate the autocorrelation function up to lag L, then let

$$(6.1) \qquad \hat{\sigma}^2_{L+1} = [(L+1) + 2 \sum_{j=1}^{L} (L+1-j)\hat{\rho}(j)]\hat{\sigma}^2 \quad .$$

If the demand history is limited, however, then the estimate $\hat{\rho}(j)$ is likely to be very unstable for higher lags. One possible solution is to consider estimating only the first-order autocorrelation and setting the others equal to zero. As a preliminary step, we have investigated the performance of the Correlation-Adjusted Power Approximation when we know the first-order autocorrelation $\rho(1)$, and substitute $\rho(j) = 0$ for all $j > 1$ in expression (5.2). A summary of its performance is provided in Table 6.1. If we compare these results with those given in Table 5.3, using all of the higher-order autocorrelations, we see that only a slight degradation in performance has occurred. These results are encouraging and suggest we may indeed only need to estimate $\rho(1)$.

Finally, the robustness of the Correlation-Adjusted Power Approximation in other types of correlated demand environments needs to be studied. For example, its performance in an environment where demand follows a Markov process will be investigated.

Table 6.1

Summary of the Performance of the Correlation-Adjusted Power Approximation for Two 72-Item Warehouse Inventory Systems Using Only the First-Order Autocorrelation

COST COMPONENT	FEW-STORES ENVIRONMENT		MANY-STORES ENVIRONMENT	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	1964 (61.9)	217 [12.4]	2204 (65.5)	369 [20.1]
BACKLOG	439 (13.8)	- 27 [- 5.9]	404 (12.0)	- 78 [-16.0]
REPLENISHMENT	768 (24.2)	-100 [-11.5]	758 (22.5)	-116 [-13.3]
TOTAL	3170 (100.0)	90 [2.9]	3366 (100.0)	176 [5.5]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

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APPENDICES

APPENDIX A

72-Item Wholesale Warehouse Inventory System,
Few-Stores Environment,
Variance-to-Mean Ratio of 9,
System Control is Best (s,S) Policy.

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 3080.4

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		26.5	31.0	42.5	45.3	54.7	30.1	33.3	36.5	16.3	23.0	28.3	32.4
	MEAN												
	4	4.3	5.1	6.9	7.4	8.9	4.9	5.5	5.9				
	8	6.1	7.1	9.8	10.4	12.6	7.0	7.7	8.3				
	12	7.5	8.8	12.1	12.8	15.5	8.6	9.4	10.3				
	16	8.6	10.1	13.7	14.7	17.8	9.7	10.8	11.9				
	LEADTIME												
	0	8.2	9.5	12.5	13.4	16.7							
	2	8.8	10.3	14.2	15.1	18.2							
	4	9.5	11.2	15.8	16.7	19.8							
	C(FIX)/C(IN)												
	32	11.8	13.9	19.6									
	64	14.7	17.1	22.9									
LEADTIME	C(FIX)/C(IN)												
0	32	3.6	4.2	5.7									
0	64	4.6	5.3	6.8									
2	32	3.9	4.7	6.5									
2	64	4.9	5.7	7.6									
4	32	4.3	5.1	7.4									
4	64	5.2	6.1	8.5									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTRCL POLICY IS BEST (S,S)

SOUPCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM= 1746.5 PERCENT OF TOTAL COST= 56.7

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		21.0	29.1	49.8	46.7	53.3	28.8	33.4	37.8	16.6	22.6	28.7	32.2
	MEAN												
	4	3.4	4.8	8.3	7.7	8.8	4.7	5.7	6.2				
	8	4.8	6.5	11.3	10.6	12.0	6.5	7.6	8.5				
	12	6.0	8.3	14.4	13.2	15.4	8.3	9.6	10.8				
	16	6.8	9.5	15.9	15.1	17.1	9.3	10.6	12.3				
	LEADTIME												
	0	6.2	8.5	14.0	13.1	15.7							
	2	7.0	9.6	16.8	15.5	17.9							
	4	7.9	11.0	19.0	18.0	19.8							
	C(FIX)/C(IN)												
	32	9.6	13.6	23.5									
	64	11.5	15.5	26.3									
LEADTIME	C(FIX)/C(IN)												
0	32	2.6	3.9	6.5									
0	64	3.5	4.6	7.5									
2	32	3.2	4.4	7.9									
2	64	3.8	5.2	8.9									
4	32	3.7	5.2	9.1									
4	64	4.2	5.8	9.9									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM= 466.3 PERCENT OF TOTAL COST= 15.1

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		38.3	35.0	26.7	49.5	50.5	28.2	32.8	39.0	16.6	23.0	28.5	31.8
	MEAN												
	4	6.6	5.7	4.4	8.6	8.0	4.5	5.7	6.4				
	8	8.9	7.9	6.2	11.2	11.8	6.7	7.4	8.9				
	12	10.6	10.2	7.7	14.4	14.1	8.1	9.3	11.1				
	16	12.2	11.2	8.4	15.3	16.5	8.9	10.4	12.5				
	LEADTIME												
	0	11.5	9.6	7.1	13.9	14.3							
	2	12.2	12.1	8.5	16.1	16.7							
	4	14.5	13.3	11.1	19.6	19.4							
	C(FIX)/C(IN)												
	32	18.9	16.9	13.7									
	64	19.4	18.1	13.0									
LEADTIME	C(FIX)/C(IN)												
0	32	5.9	4.5	3.5									
0	64	5.6	5.1	3.6									
2	32	5.9	5.8	4.3									
2	64	6.3	6.3	4.2									
4	32	7.0	6.6	5.9									
4	64	7.5	6.7	5.2									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM= 867.7 PERCENT OF TOTAL COST= 28.2

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		31.2	32.7	36.2	40.2	59.8	33.9	33.5	32.6	15.5	23.9	27.5	33.1
	MEAN												
	4	4.8	5.1	5.5	5.9	9.5	5.5	4.9	5.0				
	8	7.3	8.0	8.6	9.6	14.3	8.0	8.1	7.8				
	12	8.7	8.9	9.9	11.2	16.3	9.3	9.1	9.0				
	16	10.3	10.6	12.2	13.4	19.7	11.1	11.3	10.8				
	LEADTIME												
	0	10.4	11.3	12.2	13.8	20.1							
	2	10.7	10.9	11.9	13.7	19.8							
	4	10.0	10.5	12.0	12.6	19.9							
	C(FIX)/C(IN)												
	32	12.4	13.1	14.7									
	64	18.8	19.6	21.4									
LEADTIME	C(FIX)/C(IN)												
0	32	4.3	4.5	5.1									
0	64	6.2	6.7	7.2									
2	32	4.3	4.5	5.0									
2	64	6.4	6.4	6.9									
4	32	3.9	4.1	4.7									
4	64	6.2	6.4	7.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.0973

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.1906	0.0926	0.0088	0.0978	0.0969	0.0964	0.0965	0.0990	0.0933	0.0980	0.0989	0.0991
MEAN													
4		0.1833	0.0886	0.0080	0.0938	0.0929	0.0896	0.0927	0.0976				
8		0.1923	0.0929	0.0087	0.0984	0.0976	0.0976	0.0966	0.0997				
12		0.1929	0.0944	0.0092	0.1001	0.0977	0.0994	0.0979	0.0993				
16		0.1938	0.0944	0.0091	0.0989	0.0994	0.0992	0.0987	0.0995				
LEADTIME													
0		0.1891	0.0914	0.0089	0.0980	0.0948							
2		0.1876	0.0934	0.0085	0.0961	0.0968							
4		0.1951	0.0931	0.0089	0.0991	0.0989							
C(FIX)/C(IN)													
32		0.1920	0.0925	0.0088									
64		0.1892	0.0927	0.0087									
LEADTIME	C(FIX)/C(IN)												
0	32	0.1948	0.0905	0.0089									
0	64	0.1834	0.0922	0.0088									
2	32	0.1860	0.0939	0.0085									
2	64	0.1891	0.0928	0.0085									
4	32	0.1952	0.0932	0.0090									
4	64	0.1951	0.0930	0.0087									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0173

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.1859	0.0756	0.0052	0.0172	0.0175	0.0147	0.0171	0.0203	0.0288	0.0200	0.0165	0.0138
MEAN													
	4	0.3200	0.1223	0.0086	0.0300	0.0277	0.0234	0.0296	0.0334				
	8	0.2159	0.0857	0.0061	0.0195	0.0205	0.0175	0.0192	0.0232				
	12	0.1720	0.0735	0.0050	0.0167	0.0163	0.0141	0.0162	0.0193				
	16	0.1478	0.0604	0.0041	0.0132	0.0144	0.0115	0.0135	0.0163				
LEADTIME													
	0	0.1682	0.0621	0.0042	0.0145	0.0149							
	2	0.1774	0.0782	0.0050	0.0167	0.0174							
	4	0.2120	0.0864	0.0065	0.0204	0.0202							
C(FIX)/C(IN)													
	32	0.1833	0.0732	0.0054									
	64	0.1885	0.0780	0.0051									
LEADTIME	C(FIX)/C(IN)												
0	32	0.1720	0.0582	0.0041									
0	64	0.1645	0.0660	0.0042									
2	32	0.1724	0.0754	0.0051									
2	64	0.1825	0.0811	0.0050									
4	32	0.2054	0.0859	0.0069									
4	64	0.2186	0.0869	0.0061									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2640

	C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	0.2461	0.2584	0.2874	0.3027	0.2252	0.2698	0.2666	0.2555	0.1611	0.2530	0.2909	0.3509
MEAN												
4	0.1545	0.1604	0.1685	0.1790	0.1433	0.1699	0.1554	0.1581				
8	0.2278	0.2564	0.2747	0.2905	0.2155	0.2546	0.2604	0.2439				
12	0.2754	0.2796	0.3178	0.3363	0.2455	0.3006	0.2939	0.2783				
16	0.3268	0.3372	0.3887	0.4050	0.2967	0.3541	0.3568	0.3417				
LEADTIME												
0	0.2485	0.2673	0.2935	0.3126	0.2269							
2	0.2541	0.2603	0.2855	0.3096	0.2236							
4	0.2358	0.2475	0.2832	0.2858	0.2252							
C(FIX)/C(IN)												
32	0.2803	0.2951	0.3327									
64	0.2119	0.2217	0.2421									

LEADTIME	C(FIX)/C(IN)			
0	32	0.2882	0.3059	0.3438
0	64	0.2088	0.2287	0.2433
2	32	0.2897	0.3023	0.3368
2	64	0.2184	0.2184	0.2342
4	32	0.2630	0.2771	0.3174
4	64	0.2086	0.2180	0.2489

APPENDIX B

72-Item Wholesale Warehouse Inventory System,
Many-Stores Environment,
Variance-to-Mean Ratio of 9,
System Control is Best (s,S) Policy.

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 3189.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		26.6	31.2	42.3	45.7	54.3	29.7	34.4	36.0	16.4	23.0	28.2	32.4
MEAN													
4		4.3	5.1	7.0	7.5	8.9	4.9	5.7	5.8				
8		6.2	7.2	9.7	10.5	12.6	6.8	8.0	8.3				
12		7.5	8.8	11.9	12.9	15.3	8.4	9.7	10.2				
16		8.6	10.1	13.7	14.9	17.5	9.6	11.1	11.7				
LEADTIME													
0		7.9	9.2	12.6	13.3	16.4							
2		9.1	10.7	14.5	15.8	18.6							
4		9.6	11.3	15.1	16.6	19.4							
C(FIX)/C(IN)													
32		11.9	14.1	19.6									
64		14.7	17.0	22.6									
LEADTIME	C(FIX)/C(IN)												
0	32	3.5	4.1	5.8									
0	64	4.4	5.1	6.8									
2	32	4.1	4.9	6.8									
2	64	5.0	5.8	7.7									
4	32	4.4	5.2	7.1									
4	64	5.2	6.1	8.0									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STCRES ENVIRCNMENT
VARIANCE/MEAN = 9

SYSTEM CCNTBCL PCLICY IS EEST (S,S)

SOUBCES OF AGGREGATE PERIOC-END INVENTOBY

OVERALL AGGREGATE FOR SYSTEM= 1835.3 PERCENT OF TOTAL COST= 57.5

	C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	21.0	29.0	50.0	46.9	53.1	26.9	35.2	37.8	16.5	22.5	28.8	32.1
MEAN												
4	3.4	4.9	8.2	7.9	8.6	4.4	6.0	6.1				
8	4.8	6.5	11.3	10.8	11.7	6.0	7.9	8.6				
12	5.9	8.4	14.5	13.4	15.5	7.9	10.1	10.9				
16	6.9	9.3	16.0	14.8	17.3	8.7	11.2	12.2				
LEADTIME												
0	5.3	7.7	13.9	12.3	14.7							
2	7.4	10.2	17.6	16.6	18.6							
4	8.2	11.2	18.5	18.1	19.8							
C(FIX)/C(IN)												
32	9.8	13.6	23.6									
64	11.2	15.5	26.4									

LEADTIME	C(FIX)/C(IN)			
0	32	2.3	3.6	6.4
0	64	3.0	4.1	7.6
2	32	3.6	4.7	8.3
2	64	3.9	5.5	9.3
[illegible]	32	3.9	5.3	8.9
[illegible]	64	4.3	5.5	9.6

AD-A068 770 NORTH CAROLINA UNIV AT CHAPEL HILL SCHOOL OF BUSINESS--ETC F/G 12/1
(S,S) INVENTORY POLICIES FOR A WHOLESALE WAREHOUSE INVENTORY SY--ETC(U)
APR 79 C R SCHULTZ, H M WAGNER, R EHRHARDT N00014-78-C-0467

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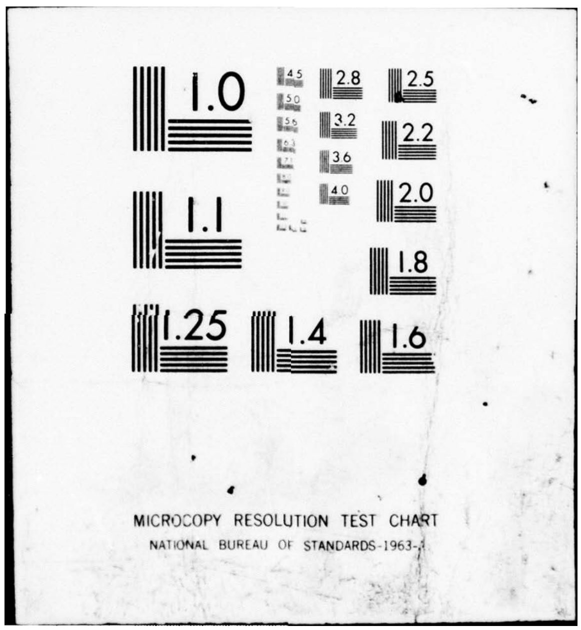
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72-ITEM WHCLESAIE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTRCI POIICY IS BEST (S,S)

SOURCES OF AGGREGATE BACKICG COST

OVERALL AGGREGATE FOR SYSTEM= 480.7 PERCENT CF TOTAL COST= 15.1

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		38.9	35.3	25.8	49.7	50.3	30.1	34.0	35.9	16.4	22.2	28.7	32.7
	MEAN												
	4	6.9	5.5	4.1	8.2	8.3	5.4	5.5	5.5				
	8	8.7	8.2	5.3	10.9	11.3	6.9	7.4	7.9				
	12	11.1	9.9	7.6	14.0	14.6	8.2	9.8	10.7				
	16	12.2	11.7	8.8	16.5	16.1	9.7	11.2	11.8				
	LEADTIME												
	0	11.3	10.5	8.3	15.2	14.9							
	2	13.1	12.1	8.8	16.4	17.6							
	4	14.5	12.6	8.7	18.1	17.8							
	C(FIX)/C(IN)												
	32	18.8	17.3	13.6									
	64	20.1	18.0	12.2									
LEADTIME	C(FIX)/C(IN)												
0	32	5.7	5.3	4.2									
0	64	5.6	5.2	4.1									
2	32	6.0	5.8	4.5									
2	64	7.1	6.3	4.2									
4	32	7.1	6.1	4.8									
4	64	7.4	6.5	3.9									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM= 873.6 PERCENT OF TOTAL COST= 27.4

	C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	31.5	33.5	35.0	40.9	59.1	35.2	32.7	32.0	16.2	24.6	26.5	32.8
MEAN												
4	4.8	5.4	5.9	6.2	10.0	5.8	5.0	5.3				
8	7.7	8.2	8.7	9.5	15.2	8.5	8.4	7.7				
12	8.7	9.0	8.8	11.2	15.3	9.4	8.6	8.4				
16	10.3	10.9	11.6	14.1	18.7	11.4	10.7	10.7				
LEADTIME												
0	11.3	11.6	12.3	14.5	20.7							
2	10.4	11.1	11.3	13.7	19.0							
4	9.8	10.8	11.5	12.7	19.4							
C(FIX)/C(IN)												
32	12.6	13.6	14.7									
64	18.9	19.9	20.3									

LEADTIME	C(FIX)/C(IN)			
0	32	4.7	4.4	5.4
0	64	6.6	7.2	6.9
2	32	4.1	4.8	4.8
2	64	6.3	6.3	6.5
4	32	3.7	4.4	4.5
4	64	6.0	6.4	7.0

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.0972

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.1881	0.0946	0.0090	0.0974	0.0971	0.0952	0.0975	0.0990	0.0925	0.0973	0.0992	0.0999
MEAN													
4		0.1766	0.0920	0.0089	0.0948	0.0902	0.0908	0.0918	0.0948				
8		0.1893	0.0938	0.0088	0.0972	0.0975	0.0936	0.0996	0.0987				
12		0.1926	0.0960	0.0091	0.0978	0.1007	0.0986	0.0977	0.1014				
16		0.1940	0.0964	0.0091	0.0996	0.1001	0.0978	0.1008	0.1009				
LEADTIME													
0		0.1811	0.0956	0.0090	0.0966	0.0938							
2		0.1898	0.0939	0.0088	0.0969	0.0981							
4		0.1936	0.0942	0.0091	0.0986	0.0993							
C(FIX)/C(IN)													
32		0.1895	0.0935	0.0091									
64		0.1868	0.0956	0.0088									
LEADTIME	C(FIX)/C(IN)												
0	32	0.1853	0.0955	0.0089									
0	64	0.1768	0.0957	0.0090									
2	32	0.1889	0.0927	0.0090									
2	64	0.1906	0.0951	0.0087									
4	32	0.1942	0.0923	0.0093									
4	64	0.1929	0.0962	0.0088									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0179

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.1948	0.0785	0.0052	0.0178	0.0180	0.0162	0.0182	0.0192	0.0294	0.0199	0.0171	0.0146
MEAN													
	4	0.3470	0.1214	0.0082	0.0292	0.0296	0.0289	0.0295	0.0297				
	8	0.2178	0.0915	0.0054	0.0195	0.0202	0.0185	0.0200	0.0212				
	12	0.1854	0.0736	0.0052	0.0167	0.0174	0.0146	0.0176	0.0190				
	16	0.1524	0.0650	0.0045	0.0148	0.0144	0.0130	0.0150	0.0158				
LEADTIME													
	0	0.1693	0.0703	0.0051	0.0163	0.0160							
	2	0.1970	0.0810	0.0053	0.0176	0.0189							
	4	0.2182	0.0842	0.0053	0.0194	0.0191							
C(FIX)/C(IN)													
	32	0.1880	0.0771	0.0055									
	64	0.2017	0.0800	0.0050									
LEADTIME	C(FIX)/C(IN)												
0	32	0.1703	0.0714	0.0051									
0	64	0.1684	0.0692	0.0050									
2	32	0.1806	0.0779	0.0055									
2	64	0.2133	0.0841	0.0051									
4	32	0.2130	0.0819	0.0059									
4	64	0.2233	0.0866	0.0047									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS BEST (S,S)

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2672

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2507	0.2676	0.2832	0.3103	0.2240	0.2831	0.2640	0.2544	0.1691	0.2583	0.2858	0.3554
MEAN													
	4	0.1542	0.1698	0.1834	0.1866	0.1516	0.1803	0.1627	0.1644				
	8	0.2435	0.2551	0.2761	0.2867	0.2298	0.2710	0.2672	0.2365				
	12	0.2779	0.2897	0.2899	0.3402	0.2315	0.3027	0.2824	0.2725				
	16	0.3271	0.3558	0.3832	0.4276	0.2832	0.3784	0.3437	0.3440				
LEADTIME													
	0	0.2740	0.2735	0.3018	0.3308	0.2354							
	2	0.2476	0.2706	0.2738	0.3115	0.2165							
	4	0.2304	0.2588	0.2739	0.2885	0.2203							
C(FIX)/C(IN)													
	32	0.2860	0.3094	0.3354									
	64	0.2153	0.2258	0.2310									
LEADTIME	C(FIX)/C(IN)												
0	32	0.3219	0.3014	0.3651									
0	64	0.2261	0.2455	0.2345									
2	32	0.2812	0.3268	0.3266									
2	64	0.2141	0.2143	0.2210									
4	32	0.2550	0.3000	0.3104									
4	64	0.2057	0.2176	0.2374									

APPENDIX C

72-Item Wholesale Warehouse Inventory System,
Few-Stores Environment,
Variance-to-Mean Ratio of 9,
System Control is Correlation-Adjusted Power Approximation.

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 3159.5

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		26.1	30.5	43.4	45.6	54.4	30.1	33.3	36.5	16.5	23.1	28.2	32.2
MEAN													
4		4.2	5.0	7.2	7.5	8.9	5.0	5.5	6.0				
8		6.0	7.1	10.0	10.5	12.6	7.0	7.7	8.4				
12		7.3	8.6	12.3	12.9	15.3	8.4	9.5	10.3				
16		8.4	9.8	13.9	14.6	17.6	9.7	10.7	11.8				
LEADTIME													
0		8.1	9.3	12.7	13.5	16.7							
2		8.7	10.2	14.5	15.2	18.1							
4		9.3	11.0	16.2	16.9	19.7							
C(FIX)/C(IN)													
32		11.5	13.7	20.3									
64		14.5	16.8	23.1									
LEADTIME	C(FIX)/C(IN)												
0	32	3.5	4.1	5.8									
0	64	4.5	5.2	6.9									
2	32	3.8	4.6	6.8									
2	64	4.8	5.6	7.7									
4	32	4.2	5.0	7.7									
4	64	5.2	6.0	8.5									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM= 1930.4 PERCENT OF TOTAL COST= 61.1

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		18.3	27.2	54.5	48.6	51.4	28.4	33.5	38.0	16.8	23.0	28.1	32.1
	MEAN												
	4	2.9	4.5	9.3	8.2	8.6	4.8	5.7	6.3				
	8	4.2	6.2	12.6	11.1	11.9	6.5	7.7	8.7				
	12	5.2	7.7	15.2	13.7	14.4	8.0	9.5	10.7				
	16	6.0	8.8	17.3	15.6	16.5	9.1	10.7	12.3				
	LEADTIME												
	0	5.3	7.9	15.3	13.6	14.9							
	2	6.1	9.1	18.3	16.4	17.2							
	4	6.9	10.2	20.9	18.7	19.3							
	C(FIX)/C(IN)												
	32	8.8	13.1	26.7									
	64	9.5	14.1	27.8									
LEADTIME	C(FIX)/C(IN)												
0	32	2.5	3.7	7.4									
0	64	2.8	4.1	7.9									
2	32	3.0	4.4	9.0									
2	64	3.1	4.7	9.3									
4	32	3.4	4.9	10.4									
4	64	3.5	5.3	10.5									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM= 461.4 PERCENT OF TOTAL COST= 14.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		46.5	39.8	13.6	43.0	57.0	30.6	32.5	36.9	17.1	24.3	27.9	30.6
	MEAN												
	4	8.3	6.9	2.0	7.0	10.2	5.5	5.4	6.2				
	8	11.3	10.1	3.0	11.0	13.3	7.4	8.0	9.0				
	12	12.8	10.7	4.4	11.9	16.0	8.1	9.4	10.3				
	16	14.2	12.2	4.2	13.1	17.5	9.5	9.7	11.4				
	LEADTIME												
	0	14.2	12.3	4.1	13.1	17.5							
	2	15.5	12.9	4.1	13.9	18.6							
	4	16.8	14.6	5.4	15.9	20.9							
	C(FIX)/C(IN)												
	32	20.2	17.3	5.5									
	64	26.4	22.5	8.1									
LEADTIME	C(FIX)/C(IN)												
0	32	6.3	5.3	1.5									
0	64	8.0	7.0	2.5									
2	32	6.6	5.5	1.8									
2	64	8.9	7.4	2.4									
4	32	7.3	6.5	2.2									
4	64	9.5	8.2	3.2									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM= 767.7 PERCENT OF TOTAL COST= 24.3

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		33.3	33.2	33.4	39.5	60.5	34.2	33.2	32.6	15.4	22.8	28.6	33.2
MEAN													
4		5.1	5.2	5.1	6.2	9.2	5.3	5.1	5.0				
8		7.6	7.5	7.7	8.9	13.9	7.8	7.5	7.4				
12		9.5	9.5	9.5	11.4	17.2	9.7	9.5	9.3				
16		11.1	11.1	11.1	13.0	20.3	11.3	11.0	10.9				
LEADTIME													
0		11.4	11.3	11.5	13.5	20.7							
2		11.1	11.0	11.1	13.1	20.0							
4		10.9	10.9	10.9	12.9	19.8							
C(FIX)/C(IN)													
32		13.2	13.1	13.2									
64		20.1	20.1	20.3									
LEADTIME	C(FIX)/C(IN)												
0	32	4.5	4.5	4.5									
0	64	6.9	6.9	7.0									
2	32	4.4	4.4	4.4									
2	64	6.7	6.7	6.7									
4	32	4.3	4.3	4.3									
4	64	6.6	6.6	6.6									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1085

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2208	0.1004	0.0042	0.0995	0.1175	0.1122	0.1080	0.1053	0.1086	0.1122	0.1057	0.1075
MEAN													
	4	0.2245	0.0979	0.0033	0.0944	0.1228	0.1113	0.1083	0.1061				
	8	0.2243	0.1081	0.0042	0.1052	0.1192	0.1166	0.1106	0.1094				
	12	0.2161	0.0961	0.0050	0.0960	0.1154	0.1089	0.1057	0.1025				
	16	0.2183	0.0996	0.0045	0.1023	0.1127	0.1118	0.1075	0.1032				
LEADTIME													
	0	0.2240	0.1078	0.0046	0.1049	0.1194							
	2	0.2248	0.0955	0.0038	0.0968	0.1192							
	4	0.2136	0.0980	0.0043	0.0967	0.1139							
C(FIX)/C(IN)													
	32	0.2023	0.0926	0.0035									
	64	0.2393	0.1083	0.0050									
LEADTIME	C(FIX)/C(IN)												
0	32	0.2079	0.1030	0.0037									
0	64	0.2401	0.1126	0.0055									
2	32	0.2010	0.0862	0.0032									
2	64	0.2486	0.1048	0.0043									
4	32	0.1980	0.0884	0.0036									
4	64	0.2291	0.1075	0.0050									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0172

	C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	0.2237	0.0851	0.0026	0.0148	0.0196	0.0158	0.0168	0.0190	0.0294	0.0209	0.0160	0.0132
MEAN												
4	0.3980	0.1466	0.0038	0.0239	0.0349	0.0285	0.0278	0.0318				
8	0.2713	0.1074	0.0029	0.0190	0.0228	0.0191	0.0205	0.0231				
12	0.2044	0.0763	0.0029	0.0136	0.0184	0.0140	0.0162	0.0178				
16	0.1708	0.0651	0.0021	0.0113	0.0150	0.0123	0.0125	0.0146				
LEADTIME												
0	0.2054	0.0788	0.0024	0.0135	0.0180							
2	0.2234	0.0828	0.0024	0.0143	0.0192							
4	0.2423	0.0937	0.0032	0.0164	0.0216							
C(FIX)/C(IN)												
32	0.1938	0.0741	0.0021									
64	0.2536	0.0961	0.0032									

LEADTIME	C(FIX)/C(IN)			
0	32	0.1809	0.0684	0.0018
0	64	0.2300	0.0892	0.0029
2	32	0.1912	0.0710	0.0020
2	64	0.2555	0.0945	0.0028
4	32	0.2093	0.0829	0.0026
4	64	0.2753	0.1045	0.0038

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2324

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2325	0.2319	0.2328	0.2632	0.2016	0.2385	0.2314	0.2273	0.1443	0.2110	0.2663	0.3079
MEAN													
4		0.1448	0.1444	0.1438	0.1661	0.1226	0.1486	0.1433	0.1411				
8		0.2110	0.2092	0.2128	0.2369	0.1851	0.2180	0.2089	0.2061				
12		0.2661	0.2661	0.2667	0.3038	0.2288	0.2723	0.2666	0.2601				
16		0.3080	0.3078	0.3080	0.3458	0.2700	0.3149	0.3069	0.3019				
LEADTIME													
0		0.2382	0.2370	0.2401	0.2698	0.2071							
2		0.2313	0.2316	0.2313	0.2626	0.2003							
4		0.2278	0.2270	0.2270	0.2571	0.1975							
C(FIX)/C(IN)													
32		0.2636	0.2629	0.2630									
64		0.2014	0.2009	0.2026									
LEADTIME	C(FIX)/C(IN)												
0	32	0.2705	0.2684	0.2706									
0	64	0.2059	0.2056	0.2057									
2	32	0.2620	0.2637	0.2620									
2	64	0.2007	0.1995	0.2007									
4	32	0.2582	0.2565	0.2565									
4	64	0.1975	0.1975	0.1975									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF TOTAL COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 2.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.9	0.9	4.8	3.2	2.0	2.6	2.5	2.6	4.1	3.0	2.1	1.9
MEAN													
	4	1.9	1.7	7.2	5.1	3.3	5.1	3.2	4.1				
	8	1.2	1.6	5.1	3.7	2.4	2.7	2.4	3.7				
	12	0.7	0.6	4.1	2.9	1.5	0.9	3.2	2.1				
	16	0.3	0.2	4.1	2.2	1.6	2.7	1.6	1.4				
LEADTIME													
	0	0.9	1.3	4.7	2.9	2.4							
	2	0.9	0.7	4.8	3.4	1.7							
	4	0.8	0.7	5.0	3.3	2.0							
C(FIX)/C(IN)													
	32	0.4	0.8	6.7									
	64	1.3	0.9	3.3									
LEADTIME	C(FIX)/C(IN)												
0	32	0.5	1.3	5.7									
0	64	1.3	1.3	3.9									
2	32	0.5	0.5	7.2									
2	64	1.2	0.8	2.8									
4	32	0.2	0.7	6.9									
4	64	1.4	0.8	3.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE PERIOD-END INVENTORY
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 10.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		-4.0	3.2	20.9	15.1	6.5	9.2	11.0	11.1	12.0	12.5	8.4	10.3
	MEAN												
	4	-4.3	2.8	24.0	17.4	7.3	12.7	10.5	12.9				
	8	-3.8	5.5	23.5	15.5	9.8	10.8	12.2	14.0				
	12	-5.2	2.9	17.3	14.4	3.2	6.3	9.2	9.2				
	16	-2.8	2.2	20.8	14.2	6.8	8.9	12.1	9.8				
	LEADTIME												
	0	-6.1	1.6	20.6	14.3	5.0							
	2	-3.5	5.2	20.4	16.4	6.4							
	4	-2.8	2.7	21.7	14.6	7.9							
	C(FIX)/C(IN)												
	32	1.6	6.3	25.7									
	64	-8.6	0.5	16.7									
LEADTIME	C(FIX)/C(IN)												
0	32	3.1	4.0	25.1									
0	64	-12.9	-0.5	16.7									
2	32	1.9	9.4	26.2									
2	64	-8.0	1.6	15.3									
4	32	0.3	5.3	25.6									
4	64	-5.6	0.4	18.0									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE BACKLOG COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -1.0

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		20.3	12.6	-49.5	-14.1	11.8	7.3	-1.8	-6.4	1.8	4.5	-3.2	-4.6
MEAN													
	4	24.4	19.9	-55.5	-20.4	26.0	21.6	-6.2	-4.9				
	8	25.7	25.3	-52.3	-2.6	11.3	8.6	6.7	-0.4				
	12	18.8	3.8	-43.0	-18.6	12.5	-0.8	0.2	-7.8				
	16	15.6	7.8	-50.3	-14.8	4.8	6.4	-7.4	-10.2				
LEADTIME													
	0	22.1	26.9	-43.4	-6.5	20.7							
	2	25.9	5.8	-52.2	-14.3	10.1							
	4	14.3	8.4	-51.4	-19.4	6.7							
C(FIX)/C(IN)													
	32	5.8	1.2	-60.5									
	64	34.5	23.2	-37.9									
LEADTIME	C(FIX)/C(IN)												
0	32	5.2	17.5	-56.8									
0	64	39.8	35.2	-30.1									
2	32	10.9	-5.8	-60.0									
2	64	40.0	16.6	-44.1									
4	32	1.9	-3.5	-63.0									
4	64	25.9	20.2	-38.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE REPLENISHMENT COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -11.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		-5.4	-10.0	-18.2	-13.1	-10.5	-10.8	-12.4	-11.4	-11.7	-15.8	-8.0	-11.3
	MEAN												
	4	-5.9	-11.2	-17.2	-7.2	-14.5	-15.2	-8.1	-11.4				
	8	-8.3	-17.2	-21.0	-18.5	-14.1	-13.2	-18.2	-16.1				
	12	-3.0	-5.5	-14.5	-9.7	-6.8	-7.9	-7.7	-8.3				
	16	-5.1	-7.7	-19.6	-14.6	-9.0	-9.2	-13.9	-10.7				
	LEADTIME												
	0	-3.3	-10.9	-16.9	-13.7	-8.7							
	2	-8.7	-10.3	-17.6	-15.2	-10.4							
	4	-4.0	-8.6	-20.1	-10.1	-12.3							
	C(FIX)/C(IN)												
	32	-6.0	-10.9	-20.9									
	64	-5.0	-9.4	-16.3									
LEADTIME	C(FIX)/C(IN)												
0	32	-6.1	-12.3	-21.3									
0	64	-1.4	-10.1	-13.8									
2	32	-9.6	-12.8	-22.2									
2	64	-8.1	-8.6	-14.3									
4	32	-1.9	-7.4	-19.2									
4	64	-5.3	-9.4	-20.6									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

BACKLOG FREQUENCY
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 11.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		15.9	8.5	-51.5	1.7	21.3	16.3	12.0	6.3	16.3	14.5	6.9	8.4
MEAN													
	4	22.4	10.5	-58.8	0.6	32.2	24.2	16.8	8.7				
	8	16.7	16.3	-51.7	6.9	22.2	19.5	14.5	9.7				
	12	12.0	1.7	-46.2	-4.1	18.2	9.6	7.9	3.3				
	16	12.7	5.5	-50.4	3.5	13.4	12.7	9.0	3.7				
LEADTIME													
	0	18.5	18.0	-48.0	7.0	25.9							
	2	19.8	2.3	-55.4	0.7	23.2							
	4	9.5	5.2	-51.3	-2.5	15.1							
C(FIX)/C(IN)													
	32	5.4	0.0	-60.0									
	64	26.5	16.9	-42.9									
LEADTIME	C(FIX)/C(IN)												
0	32	6.8	13.9	-58.4									
0	64	30.9	22.1	-37.6									
2	32	8.0	-8.2	-61.8									
2	64	31.5	12.9	-49.1									
4	32	1.5	-5.2	-60.0									
4	64	17.5	15.6	-42.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

WEIGHTED PROPORTION OF DEMAND BACKLOGGED
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -1.0

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		20.3	12.6	-49.5	-14.1	11.8	7.3	-1.8	-6.4	1.8	4.5	-3.2	-4.6
	MEAN												
	4	24.4	19.9	-55.5	-20.4	26.0	21.6	-6.2	-4.9				
	8	25.7	25.3	-52.3	-2.6	11.3	8.6	6.7	-0.4				
	12	18.8	3.8	-43.0	-18.6	12.5	-0.8	0.2	-7.8				
	16	15.6	7.8	-50.3	-14.8	4.8	6.4	-7.4	-10.2				
	LEADTIME												
	0	22.1	26.9	-43.4	-6.5	20.7							
	2	25.9	5.8	-52.2	-14.3	10.1							
	4	14.3	8.4	-51.4	-19.4	6.7							
	C(FIX)/C(IN)												
	32	5.8	1.2	-60.5									
	64	34.5	23.2	-37.9									
LEADTIME	C(FIX)/C(IN)												
0	32	5.2	17.5	-56.8									
0	64	39.8	35.2	-30.1									
2	32	10.9	-5.8	-60.0									
2	64	40.0	16.6	-44.1									
4	32	1.9	-3.5	-63.0									
4	64	25.9	20.2	-38.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
FEW-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

REPLENISHMENT FREQUENCY
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -12.0

	C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	-5.6	-10.3	-19.0	-13.1	-10.5	-11.6	-13.2	-11.0	-10.4	-16.6	-8.5	-12.2
MEAN												
4	-6.3	-10.0	-14.6	-7.2	-14.5	-12.5	-7.8	-10.8				
8	-7.4	-18.4	-22.5	-18.5	-14.1	-14.4	-19.8	-15.5				
12	-3.4	-4.8	-16.1	-9.7	-6.8	-9.4	-9.3	-6.5				
16	-5.8	-8.7	-20.8	-14.6	-9.0	-11.1	-14.0	-11.7				
LEADTIME												
0	-4.1	-11.3	-18.2	-13.7	-8.7							
2	-8.9	-11.0	-19.0	-15.2	-10.4							
4	-3.4	-8.3	-19.8	-10.1	-12.3							
C(FIX)/C(IN)												
32	-6.0	-10.9	-20.9									
64	-5.0	-9.4	-16.3									

LEADTIME	C(FIX)/C(IN)			
0	32	-6.1	-12.3	-21.3
0	64	-1.4	-10.1	-13.8
2	32	-9.6	-12.8	-22.2
2	64	-8.1	-8.6	-14.3
4	32	-1.9	-7.4	-19.2
4	64	-5.3	-9.4	-20.6

APPENDIX D

72-Item Wholesale Warehouse Inventory System,
Many-Stores Environment,
Variance-to-Mean Ratio of 9,
System Control is Correlation-Adjusted Power Approximation.

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM= 3298.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		25.9	30.4	43.8	46.0	54.0	29.1	34.5	36.4	16.7	23.1	28.0	32.2
	MEAN												
	4	4.3	5.0	7.4	7.7	9.0	4.8	5.8	6.1				
	8	6.0	7.0	10.2	10.6	12.5	6.7	7.9	8.4				
	12	7.3	8.6	12.2	12.8	15.2	8.2	9.7	10.2				
	16	8.4	9.8	14.0	14.9	17.3	9.3	11.1	11.8				
	LEADTIME												
	0	7.7	8.9	12.5	13.1	16.0							
	2	8.9	10.4	15.2	15.9	18.5							
	4	9.4	11.0	16.1	17.0	19.5							
	C(FIX)/C(IN)												
	32	11.6	13.8	20.7									
	64	14.3	16.6	23.1									
LEADTIME	C(FIX)/C(IN)												
0	32	3.4	4.0	5.8									
0	64	4.3	5.0	6.7									
2	32	4.0	4.8	7.2									
2	64	4.9	5.6	8.0									
4	32	4.2	5.0	7.7									
4	64	5.1	6.0	8.4									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE PERIOD-END INVENTORY

OVERALL AGGREGATE FOR SYSTEM= 2080.0 PERCENT OF TOTAL COST= 63.1

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		18.2	27.0	54.7	48.7	51.3	26.4	35.1	38.5	17.3	23.2	27.6	32.0
	MEAN												
	4	3.1	4.6	9.6	8.4	8.9	4.7	6.0	6.6				
	8	4.1	6.3	12.8	11.3	11.8	6.0	8.2	9.0				
	12	5.0	7.5	15.1	13.4	14.2	7.3	9.7	10.6				
	16	6.0	8.7	17.3	15.6	16.4	8.4	11.2	12.3				
	LEADTIME												
	0	4.9	7.3	14.2	12.6	13.8							
	2	6.3	9.4	19.3	17.1	18.0							
	4	7.0	10.3	21.2	19.0	19.5							
	C(FIX)/C(IN)												
	32	8.8	13.0	26.9									
	64	9.4	14.0	27.9									
LEADTIME	C(FIX)/C(IN)												
0	32	2.3	3.5	6.8									
0	64	2.6	3.9	7.4									
2	32	3.1	4.5	9.5									
2	64	3.3	4.9	9.8									
4	32	3.4	5.0	10.5									
4	64	3.6	5.3	10.7									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE BACKLOG COST

OVERALL AGGREGATE FOR SYSTEM= 454.1 PERCENT OF TOTAL COST= 13.8

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		48.3	40.7	10.9	44.9	55.1	32.6	33.8	33.5	15.1	22.8	30.3	31.8
MEAN													
4		7.6	6.4	1.0	7.0	8.1	4.5	5.5	5.1				
8		11.6	9.1	2.2	9.4	13.4	8.3	7.3	7.2				
12		14.4	12.3	3.5	13.4	16.9	9.7	10.4	10.2				
16		14.7	12.8	4.2	15.0	16.7	10.2	10.6	11.0				
LEADTIME													
0		14.1	12.2	6.3	14.5	18.1							
2		16.7	14.1	3.1	15.6	18.2							
4		17.5	14.5	1.6	14.7	18.8							
C(FIX)/C(IN)													
32		21.7	18.3	4.9									
64		26.7	22.4	6.0									
LEADTIME	C(FIX)/C(IN)												
0	32	6.3	5.3	2.9									
0	64	7.8	6.9	3.4									
2	32	7.6	6.7	1.4									
2	64	9.1	7.4	1.7									
4	32	7.7	6.3	0.7									
4	64	9.8	8.2	0.9									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED FA

SOURCES OF AGGREGATE REPLENISHMENT COST

OVERALL AGGREGATE FOR SYSTEM= 764.6 PERCENT OF TOTAL COST= 23.2

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		33.4	33.2	33.4	39.5	60.5	34.4	33.0	32.6	16.0	23.0	27.9	33.1
MEAN													
4		5.4	5.3	5.3	6.4	9.6	5.6	5.2	5.2				
8		7.6	7.6	7.7	9.1	13.8	7.8	7.6	7.6				
12		9.3	9.2	9.3	11.0	16.9	9.7	9.2	9.1				
16		11.1	11.0	11.0	12.9	20.2	11.4	11.0	10.8				
LEADTIME													
0		11.5	11.4	11.5	13.6	20.8							
2		11.0	10.9	11.1	12.9	20.0							
4		10.9	10.9	10.8	12.9	19.7							
C(FIX)/C(IN)													
32		13.2	13.1	13.2									
64		20.2	20.1	20.3									
LEADTIME	C(FIX)/C(IN)												
0	32	4.6	4.5	4.5									
0	64	6.9	6.9	7.0									
2	32	4.3	4.2	4.4									
2	64	6.7	6.6	6.7									
4	32	4.3	4.3	4.2									
4	64	6.6	6.6	6.6									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

BACKLOG FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.1036

		C(CUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2117	0.0958	0.0034	0.0990	0.1083	0.1027	0.1046	0.1036	0.0980	0.1052	0.1082	0.1031
MEAN													
4		0.1995	0.0924	0.0021	0.0986	0.0974	0.0897	0.1039	0.1004				
8		0.2200	0.0923	0.0034	0.0999	0.1106	0.1099	0.1050	0.1008				
12		0.2198	0.1005	0.0042	0.1012	0.1152	0.1060	0.1093	0.1092				
16		0.2075	0.0979	0.0038	0.0962	0.1100	0.1052	0.1001	0.1040				
LEADTIME													
0		0.2078	0.0944	0.0059	0.0964	0.1090							
2		0.2135	0.0976	0.0027	0.1033	0.1059							
4		0.2138	0.0955	0.0016	0.0972	0.1100							
C(FIX)/C(IN)													
32		0.2082	0.0861	0.0026									
64		0.2152	0.1055	0.0042									
LEADTIME	C(FIX)/C(IN)												
0	32	0.2063	0.0783	0.0045									
0	64	0.2093	0.1104	0.0072									
2	32	0.2124	0.0953	0.0021									
2	64	0.2146	0.0959	0.0032									
4	32	0.2059	0.0847	0.0010									
4	64	0.2217	0.1062	0.0021									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

WEIGHTED PROPORTION OF DEMAND BACKLOGGED

OVERALL AGGREGATE FOR SYSTEM= 0.0169

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2286	0.0856	0.0021	0.0152	0.0186	0.0165	0.0172	0.0170	0.0255	0.0193	0.0171	0.0134
MEAN													
4		0.3597	0.1356	0.0020	0.0237	0.0273	0.0226	0.0279	0.0261				
8		0.2739	0.0555	0.0021	0.0159	0.0226	0.0211	0.0185	0.0183				
12		0.2272	0.0865	0.0023	0.0151	0.0190	0.0163	0.0176	0.0172				
16		0.1743	0.0675	0.0020	0.0127	0.0141	0.0129	0.0135	0.0139				
LEADTIME													
0		0.2006	0.0769	0.0036	0.0147	0.0183							
2		0.2369	0.0886	0.0018	0.0158	0.0185							
4		0.2484	0.0913	0.0009	0.0149	0.0190							
C(FIX)/C(IN)													
32		0.2050	0.0770	0.0019									
64		0.2523	0.0942	0.0023									
LEADTIME	C(FIX)/C(IN)												
0	32	0.1800	0.0673	0.0033									
0	64	0.2211	0.0866	0.0039									
2	32	0.2150	0.0840	0.0016									
2	64	0.2588	0.0933	0.0020									
4	32	0.2199	0.0797	0.0008									
4	64	0.2770	0.1029	0.0010									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

REPLENISHMENT FREQUENCY

OVERALL AGGREGATE FOR SYSTEM= 0.2314

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		0.2320	0.2303	0.2318	0.2618	0.2009	0.2392	0.2286	0.2264	0.1488	0.2132	0.2578	0.3057
	MEAN												
	4	0.1508	0.1474	0.1481	0.1698	0.1277	0.1580	0.1445	0.1438				
	8	0.2125	0.2126	0.2144	0.2427	0.1836	0.2153	0.2125	0.2117				
	12	0.2581	0.2562	0.2592	0.2916	0.2240	0.2696	0.2536	0.2503				
	16	0.3067	0.3049	0.3056	0.3431	0.2683	0.3140	0.3036	0.2996				
	LEADTIME												
	0	0.2391	0.2386	0.2398	0.2716	0.2068							
	2	0.2291	0.2259	0.2306	0.2576	0.1995							
	4	0.2278	0.2262	0.2250	0.2562	0.1965							
	C(FIX)/C(IN)												
	32	0.2631	0.2605	0.2619									
	64	0.2010	0.2000	0.2018									
LEADTIME	C(FIX)/C(IN)												
0	32	0.2721	0.2713	0.2715									
0	64	0.2061	0.2060	0.2082									
2	32	0.2585	0.2538	0.2605									
2	64	0.1998	0.1980	0.2007									
4	32	0.2585	0.2565	0.2536									
4	64	0.1970	0.1960	0.1964									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF TOTAL COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 3.4

	C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
	4	9	99	32	64	0	2	4	4	8	12	16
	0.7	0.7	7.1	4.1	2.8	1.4	3.7	4.8	5.2	3.6	2.9	2.9
MEAN												
4	1.1	2.0	10.1	6.7	3.9	1.8	5.2	8.0				
8	0.4	-0.1	8.5	4.5	2.9	2.2	3.2	5.2				
12	1.0	0.9	5.5	3.1	2.7	1.1	3.5	3.7				
16	0.5	0.5	6.1	3.5	2.3	0.9	3.5	3.8				
LEADTIME												
0	0.8	0.8	2.3	1.7	1.2							
2	0.5	0.5	8.0	4.5	3.0							
4	0.9	0.9	10.2	5.7	4.0							
C(FIX)/C(IN)												
32	0.5	0.7	8.8									
64	0.9	0.7	5.7									

LEADTIME	C(FIX)/C(IN)			
0	32	0.4	0.7	3.3
0	64	1.1	0.8	1.5
2	32	0.5	0.7	9.6
2	64	0.5	0.3	6.7
4	32	0.6	0.7	12.6
4	64	1.1	1.0	8.2

72-ITEM WHOLESALE WAREHOUSE INVENTCRY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SCUBCES OF AGGREGATE PERIOD-END INVENTORY
(% EXCESS CVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 13.3

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		-1.7	5.7	24.1	17.5	9.6	11.1	13.0	15.2	18.5	16.7	8.4	12.7
MEAN													
4		2.1	7.4	32.0	19.4	17.7	21.0	13.4	21.8				
8		-1.5	9.4	28.6	18.7	14.8	13.9	17.6	17.9				
12		-4.2	1.2	17.7	13.5	3.9	4.9	8.7	10.6				
16		-1.5	6.3	22.6	19.1	7.3	9.8	13.4	14.2				
LEADTIME													
0		3.5	8.1	15.7	16.5	6.6							
2		-3.4	5.2	24.4	16.8	9.6							
4		-3.5	4.5	30.1	18.9	11.9							
C(FIX)/C(IN)													
32		1.4	8.8	29.2									
64		-4.4	2.5	19.5									
LEADTIME	C(FIX)/C(IN)												
0	32	12.2	9.4	22.0									
0	64	-3.3	7.C	10.4									
2	32	-2.9	9.6	29.2									
2	64	-3.9	1.4	20.1									
4	32	-1.0	7.7	34.3									
4	64	-5.8	1.5	26.2									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN ≈ 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SCORES OF AGGREGATE BACKLOG COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -5.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		17.4	9.1	-60.0	-14.6	3.4	2.2	-6.0	-11.7	-13.3	-3.1	-0.2	-8.1
	MEAN												
	4	3.7	11.7	-75.9	-18.9	-7.7	-21.9	-5.7	-12.4				
	8	25.7	4.4	-61.5	-18.4	11.8	14.2	-7.6	-13.8				
	12	22.6	17.5	-56.3	-9.7	8.9	11.6	0.3	-9.6				
	16	14.4	3.9	-55.0	-14.0	-2.0	-0.7	-10.5	-11.9				
	LEADTIME												
	0	18.4	9.4	-28.8	-9.7	14.4							
	2	20.3	9.5	-66.6	-9.9	-2.3							
	4	13.9	8.4	-83.2	-22.9	-0.3							
	C(FIX)/C(IN)												
	32	9.0	-0.1	-65.7									
	64	25.1	17.9	-53.7									
LEADTIME	C(FIX)/C(IN)												
0	32	5.7	-5.8	-35.4									
0	64	31.3	25.1	-21.9									
2	32	19.1	7.9	-71.2									
2	64	21.3	10.9	-61.7									
4	32	3.2	-2.7	-86.7									
4	64	24.0	18.8	-78.9									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

SOURCES OF AGGREGATE REPLENISHMENT COST
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -12.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		-7.2	-13.2	-16.5	-15.6	-10.3	-14.5	-11.8	-10.9	-13.2	-18.3	-7.9	-11.5
	MEAN												
	4	-2.4	-13.7	-21.5	-9.0	-15.8	-16.4	-8.9	-13.7				
	8	-13.1	-18.4	-22.7	-15.3	-20.1	-20.4	-20.2	-13.8				
	12	-6.5	-9.8	-7.4	-14.3	-3.2	-10.5	-7.4	-5.5				
	16	-5.7	-11.8	-16.3	-19.8	-5.2	-12.5	-10.2	-11.7				
	LEADTIME												
	0	-11.6	-13.8	-17.9	-17.9	-12.1							
	2	-7.2	-14.0	-13.9	-17.3	-7.8							
	4	-2.1	-11.8	-17.7	-11.2	-10.8							
	C(FIX)/C(IN)												
	32	-8.0	-15.8	-21.9									
	64	-6.7	-11.4	-12.6									
LEADTIME	C(FIX)/C(IN)												
0	32	-15.4	-10.0	-26.5									
0	64	-8.8	-16.1	-11.2									
2	32	-8.1	-22.3	-20.2									
2	64	-6.7	-7.6	-9.2									
4	32	1.4	-14.5	-18.3									
4	64	-4.2	-9.9	-17.2									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

BACKLOG FREQUENCY
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= 6.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		12.5	1.3	-62.4	1.7	11.5	7.8	7.3	4.7	6.0	8.1	9.0	3.2
	MEAN												
	4	13.0	0.5	-76.5	4.0	8.0	-1.2	13.2	5.8				
	8	16.2	-1.5	-61.5	2.8	13.5	17.4	5.4	2.1				
	12	14.1	4.7	-54.0	3.5	14.4	7.5	11.9	7.7				
	16	7.0	1.6	-58.0	-3.4	9.9	7.5	-0.7	3.1				
	LEADTIME												
	0	14.8	-1.3	-34.5	-0.2	16.1							
	2	12.5	4.0	-69.9	6.6	7.9							
	4	10.4	1.3	-82.7	-1.4	10.8							
	C(FIX)/C(IN)												
	32	9.9	-7.9	-71.7									
	64	15.2	10.3	-52.9									
LEADTIME	C(FIX)/C(IN)												
0	32	11.3	-17.5	-49.4									
0	64	18.3	15.4	-19.7									
2	32	12.4	2.5	-76.1									
2	64	12.6	5.1	-63.5									
4	32	6.0	-8.2	-88.8									
4	64	14.9	10.4	-76.3									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

WEIGHTED PROPORTION OF DEMAND BACKLOGGED
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -5.5

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		17.4	9.1	-60.0	-14.6	3.4	2.2	-6.0	-11.7	-13.3	-3.1	-0.2	-8.1
	MEAN												
	4	3.7	11.7	-75.9	-18.9	-7.7	-21.9	-5.7	-12.4				
	8	25.7	4.4	-61.5	-18.4	11.8	14.2	-7.6	-13.8				
	12	22.6	17.5	-56.3	-9.7	8.9	11.6	0.3	-9.6				
	16	14.4	3.9	-55.0	-14.0	-2.0	-0.7	-10.5	-11.9				
	LEADTIME												
	0	18.4	9.4	-28.8	-9.7	14.4							
	2	20.3	9.5	-66.6	-9.9	-2.3							
	4	13.9	8.4	-83.2	-22.9	-0.3							
	C(FIX)/C(IN)												
	32	9.0	-0.1	-65.7									
	64	25.1	17.9	-53.7									
LEADTIME	C(FIX)/C(IN)												
0	32	5.7	-5.8	-35.4									
0	64	31.3	25.1	-21.9									
2	32	19.1	7.9	-71.2									
2	64	21.3	10.9	-61.7									
4	32	3.2	-2.7	-86.7									
4	64	24.0	18.8	-78.9									

72-ITEM WHOLESALE WAREHOUSE INVENTORY SYSTEM
MANY-STORES ENVIRONMENT
VARIANCE/MEAN = 9

SYSTEM CONTROL POLICY IS CORRELATION ADJUSTED PA

REPLENISHMENT FREQUENCY
(% EXCESS OVER BEST)

OVERALL AGGREGATE FOR SYSTEM= -13.4

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
		-7.4	-14.0	-18.1	-15.6	-10.3	-15.5	-13.4	-11.0	-12.0	-17.5	-9.8	-14.0
	MEAN												
	4	-2.2	-13.2	-19.3	-9.0	-15.8	-12.4	-11.2	-12.5				
	8	-12.7	-16.7	-22.3	-15.3	-20.1	-20.6	-20.5	-10.5				
	12	-7.2	-11.6	-10.6	-14.3	-3.2	-10.9	-10.2	-8.1				
	16	-6.2	-14.3	-20.2	-19.8	-5.2	-17.0	-11.7	-12.9				
	LEADTIME												
	0	-12.7	-12.7	-20.5	-17.9	-12.1							
	2	-7.5	-16.5	-15.8	-17.3	-7.8							
	4	-1.1	-12.6	-17.8	-11.2	-10.8							
	C(FIX)/C(IN)												
	32	-8.0	-15.8	-21.9									
	64	-6.7	-11.4	-12.6									
LEADTIME	C(FIX)/C(IN)												
0	32	-15.4	-10.0	-26.5									
0	64	-8.8	-16.1	-11.2									
2	32	-8.1	-22.3	-20.2									
2	64	-6.7	-7.6	-9.2									
4	32	1.4	-14.5	-18.3									
4	64	-4.2	-9.9	-17.2									